F07BRF (CGBTRF/ZGBTRF) – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

F07BRF (CGBTRF/ZGBTRF) computes the LU factorization of a complex m by n band matrix.

2 Specification

```
SUBROUTINE F07BRF(M, N, KL, KU, AB, LDAB, IPIV, INFO)
ENTRY      cgbtrf(M, N, KL, KU, AB, LDAB, IPIV, INFO)
INTEGER           M, N, KL, KU, LDAB, IPIV(*), INFO
complex           AB(LDAB,*)
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine forms the LU factorization of a complex m by n band matrix A using partial pivoting, with row interchanges. Usually $m = n$, and then, if A has k_l non-zero sub-diagonals and k_u non-zero super-diagonals, the factorization has the form $A = PLU$ where:

- P is a permutation matrix;
- L is a lower triangular matrix with unit diagonal elements and at most k_l non-zero elements in each column; and
- U is an upper triangular band matrix with $k_l + k_u$ super-diagonals.

Note that L is not a band matrix, but the non-zero elements of L can be stored in the same space as the sub-diagonal elements of A. U is a band matrix but with k_l additional super-diagonals compared with A. These additional super-diagonals are created by the row interchanges.

4 References

[1] Golub G H and van Loan C F (1996) *Matrix Computations* Johns Hopkins University Press (3rd Edition), Baltimore

5 Parameters

1: M — INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraint: $M \geq 0$.

2: N — INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraint: $N \geq 0$.

3: KL — INTEGER *Input*

On entry: k_l, the number of sub-diagonals within the band of A.

Constraint: $KL \geq 0$.

4: KU — INTEGER *Input*

On entry: k_u, the number of super-diagonals within the band of A.

Constraint: $KU \geq 0$.

5: AB(LDAB,$*$) — ***complex*** array *Input/Output*

Note: the second dimension of the array AB must be at least max(1,N).

On entry: the m by n band matrix A, stored in rows (k_l+1) to $(2k_l+k_u+1)$; the first k_l rows need not be set. More precisely, element $a_{i,j}$ must be stored in AB$(k_l+k_u+i-j+1,j)$ for $\max(j-k_u,1) \leq i \leq \min(j+k_l,m)$.

On exit: A is overwritten by details of the factorization: the upper triangular band matrix U with k_l+k_u super-diagonals is stored in rows 1 to (k_l+k_u+1) of the array, and the multipliers used to form the matrix L are stored in rows (k_l+k_u+2) to $(2k_l+k_u+1)$.

6: LDAB — INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07BRF (CGBTRF/ZGBTRF) is called.

Constraint: LDAB $\geq 2\times$ KL + KU + 1.

7: IPIV($*$) — INTEGER array *Output*

Note: the dimension of the array IPIV must be at least max(1,min(M,N)).

On exit: the pivot indices. Row i of the matrix A was interchanged with row IPIV(i), for $i = 1,2,\ldots,\min(m,n)$.

8: INFO — INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $u_{i,i}$ is exactly zero. The factorization has been completed but the factor U is exactly singular, and division by zero will occur if it is subsequently used to solve a system of linear equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A+E$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll \min(m,n)$.

8 Further Comments

The total number of real floating-point operations varies between approximately $8nk_l(k_u+1)$ and $8nk_l(k_l+k_u+1)$, depending on the interchanges, assuming $m = n \gg k_l$ and $n \gg k_u$.

A call to this routine may be followed by calls to the routines:

F07BSF (CGBTRS/ZGBTRS) to solve $AX = B$, $A^TX = B$ or $A^HX = B$;

F07BUF (CGBCON/ZGBCON) to estimate the condition number of A.

The real analogue of this routine is F07BDF (SGBTRF/DGBTRF).

9 Example

To compute the LU factorization of the matrix A, where

$$A = \begin{pmatrix} -1.65+2.26i & -2.05-0.85i & 0.97-2.84i & 0.00+0.00i \\ 0.00+6.30i & -1.48-1.75i & -3.99+4.01i & 0.59-0.48i \\ 0.00+0.00i & -0.77+2.83i & -1.06+1.94i & 3.33-1.04i \\ 0.00+0.00i & 0.00+0.00i & 4.48-1.09i & -0.46-1.72i \end{pmatrix}.$$

Here A is treated as a band matrix with 1 sub-diagonal and 2 super-diagonals.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F07BRF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, KLMAX, KUMAX, LDAB
      PARAMETER        (MMAX=8,NMAX=8,KLMAX=8,KUMAX=8,
     +                 LDAB=2*KLMAX+KUMAX+1)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, K, KL, KU, M, N
*     .. Local Arrays ..
      complex          AB(LDAB,NMAX)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         cgbtrf, X04DFF
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07BRF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N, KL, KU
      IF (M.LE.MMAX .AND. N.LE.NMAX .AND. KL.LE.KLMAX .AND. KU.LE.KUMAX)
     +    THEN
*
*        Read A from data file
*
         K = KL + KU + 1
         READ (NIN,*) ((AB(K+I-J,J),J=MAX(I-KL,1),MIN(I+KU,N)),I=1,M)
*
*        Factorize A
*
         CALL cgbtrf(M,N,KL,KU,AB,LDAB,IPIV,INFO)
*
*        Print details of factorization
*
         WRITE (NOUT,*)
         IFAIL = 0
         CALL X04DFF(M,N,KL,KL+KU,AB,LDAB,'Bracketed','F7.4',
     +               'Details of factorization','Integer',RLABS,
     +               'Integer',CLABS,80,0,IFAIL)
*
```

```
*        Print pivot indices
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'IPIV'
         WRITE (NOUT,99999) (IPIV(I),I=1,MIN(M,N))
*
         IF (INFO.NE.0) WRITE (NOUT,*) 'The factor U is singular'
*
      END IF
      STOP
*
99999 FORMAT ((1X,I12,3I18))
      END
```

9.2 Program Data

```
F07BRF Example Program Data
  4  4  1  2                                      :Values of M, N, KL and KU
 (-1.65, 2.26) (-2.05,-0.85) ( 0.97,-2.84)
 ( 0.00, 6.30) (-1.48,-1.75) (-3.99, 4.01) ( 0.59,-0.48)
               (-0.77, 2.83) (-1.06, 1.94) ( 3.33,-1.04)
                             ( 4.48,-1.09) (-0.46,-1.72)  :End of matrix A
```

9.3 Program Results

```
 F07BRF Example Program Results

 Details of factorization
                   1                  2                  3                  4
 1  ( 0.0000, 6.3000) (-1.4800,-1.7500) (-3.9900, 4.0100) ( 0.5900,-0.4800)
 2  ( 0.3587, 0.2619) (-0.7700, 2.8300) (-1.0600, 1.9400) ( 3.3300,-1.0400)
 3                    ( 0.2314, 0.6358) ( 4.9303,-3.0086) (-1.7692,-1.8587)
 4                                      ( 0.7604, 0.2429) ( 0.4338, 0.1233)

 IPIV
           2                 3                 3                 4
```