F07HGF (SPBCON/DPBCON) – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

F07HGF (SPBCON/DPBCON) estimates the condition number of a real symmetric positive-definite band matrix A, where A has been factorized by F07HDF (SPBTRF/DPBTRF).

2 Specification

```
 SUBROUTINE F07HGF(UPLO, N, KD, AB, LDAB, ANORM, RCOND, WORK,
1                  IWORK, INFO)
 ENTRY      spbcon(UPLO, N, KD, AB, LDAB, ANORM, RCOND, WORK,
1                  IWORK, INFO)
 INTEGER           N, KD, LDAB, IWORK(*), INFO
 real              AB(LDAB,*), ANORM, RCOND, WORK(*)
 CHARACTER*1       UPLO
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine estimates the condition number (in the 1-norm) of a real symmetric positive-definite band matrix A:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The routine should be preceded by a call to F06REF to compute $\|A\|_1$ and a call to F07HDF (SPBTRF/DPBTRF) to compute the Cholesky factorization of A. The routine then uses Higham's implementation of Hager's method [1] to estimate $\|A^{-1}\|_1$.

4 References

[1] Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: UPLO — CHARACTER*1 *Input*

On entry: indicates whether A has been factorized as U^TU or LL^T as follows:

if UPLO = 'U', then $A = U^TU$, where U is upper triangular;
if UPLO = 'L', then $A = LL^T$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N — INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: KD — INTEGER *Input*

On entry: k, the number of super-diagonals or sub-diagonals of the matrix A.

Constraint: KD ≥ 0.

4: AB(LDAB,$*$) — ***real*** array *Input*

Note: the second dimension of the array AB must be at least max(1,N).

On entry: the Cholesky factor of A, as returned by F07HDF (SPBTRF/DPBTRF).

5: LDAB — INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07HGF (SPBCON/DPBCON) is called.

Constraint: LDAB $\geq$ KD + 1.

6: ANORM — ***real*** *Input*

On entry: the 1-norm of the **original** matrix A, which may be computed by calling F06REF. ANORM must be computed either **before** calling F07HDF (SPBTRF/DPBTRF) or else from a copy of the original matrix A.

Constraint: ANORM ≥ 0.0.

7: RCOND — ***real*** *Output*

On exit: an estimate of the reciprocal of the condition number of A. RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than ***machine precision***, then A is singular to working precision.

8: WORK($*$) — ***real*** array *Workspace*

Note: the dimension of the array WORK must be at least max(1,3$*$N).

9: IWORK($*$) — INTEGER array *Workspace*

Note: the dimension of the array IWORK must be at least max(1,N).

10: INFO — INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to this routine involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $4nk$ floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to F07HEF (SPBTRS/DPBTRS) with 1 right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this routine is F07HUF (CPBCON/ZPBCON).

9 Example

To estimate the condition number in the 1-norm (or infinity-norm) of the matrix A, where

$$A = \begin{pmatrix} 5.49 & 2.68 & 0.00 & 0.00 \\ 2.68 & 5.63 & -2.39 & 0.00 \\ 0.00 & -2.39 & 2.60 & -2.22 \\ 0.00 & 0.00 & -2.22 & 5.17 \end{pmatrix}.$$

Here A is symmetric and positive-definite, and is treated as a band matrix, which must first be factorized by F07HDF (SPBTRF/DPBTRF). The true condition number in the 1-norm is 74.15.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F07HGF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, KDMAX, LDAB
      PARAMETER        (NMAX=8,KDMAX=8,LDAB=KDMAX+1)
*     .. Local Scalars ..
      real             ANORM, RCOND
      INTEGER          I, INFO, J, KD, N
      CHARACTER        UPLO
*     .. Local Arrays ..
      real             AB(LDAB,NMAX), WORK(3*NMAX)
      INTEGER          IWORK(NMAX)
*     .. External Functions ..
      real             F06REF, X02AJF
      EXTERNAL         F06REF, X02AJF
*     .. External Subroutines ..
      EXTERNAL         spbcon, spbtrf
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07HGF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, KD
      IF (N.LE.NMAX .AND. KD.LE.KDMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            DO 20 I = 1, N
               READ (NIN,*) (AB(KD+1+I-J,J),J=I,MIN(N,I+KD))
   20       CONTINUE
         ELSE IF (UPLO.EQ.'L') THEN
            DO 40 I = 1, N
               READ (NIN,*) (AB(1+I-J,J),J=MAX(1,I-KD),I)
   40       CONTINUE
         END IF
*
```

```
*        Compute norm of A
*
         ANORM = F06REF('1-norm',UPLO,N,KD,AB,LDAB,WORK)
*
*        Factorize A
*
         CALL spbtrf(UPLO,N,KD,AB,LDAB,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Estimate condition number
*
            CALL spbcon(UPLO,N,KD,AB,LDAB,ANORM,RCOND,WORK,IWORK,INFO)
*
            IF (RCOND.GE.X02AJF()) THEN
               WRITE (NOUT,99999) 'Estimate of condition number =',
     +           1.0e0/RCOND
            ELSE
               WRITE (NOUT,*) 'A is singular to working precision'
            END IF
         ELSE
            WRITE (NOUT,*) 'A is not positive-definite'
         END IF
      END IF
      STOP
*
99999 FORMAT (1X,A,1P,e10.2)
      END
```

9.2 Program Data

```
F07HGF Example Program Data
  4  1                        :Values of N and KD
  'L'                         :Value of UPLO
  5.49
  2.68   5.63
        -2.39   2.60
                -2.22   5.17   :End of matrix A
```

9.3 Program Results

```
 F07HGF Example Program Results

 Estimate of condition number =  7.42E+01
```