G01FFF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

G01FFF returns the deviate associated with the given lower tail probability of the gamma distribution, via the routine name.

2 Specification

```
real FUNCTION G01FFF(P, A, B, TOL, IFAIL)
INTEGER              IFAIL
real                 P, A, B, TOL
```

3 Description

The deviate, g_p, associated with the lower tail probability, p, of the gamma distribution with shape parameter α and scale parameter β, is defined as the solution to:

$$P(G \leq g_p : \alpha, \beta) = p = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \int_0^{g_p} e^{-G/\beta} G^{\alpha-1} dG, \qquad 0 \leq g_p < \infty \; ; \; \alpha, \beta > 0.$$

The method used is described by Best and Roberts [1] making use of the relationship between the gamma distribution and the χ^2 distribution.

Let $y = 2\frac{g_p}{\beta}$. The required y is found from the Taylor series expansion

$$y = y_0 + \sum_r \frac{C_r(y_0)}{r!} \left(\frac{E}{\phi(y_0)}\right)^r$$

where y_0 is a starting approximation

$$C_1(u) = 1$$

$$C_{r+1}(u) = \left(r\Psi + \frac{d}{du}\right) C_r(u)$$

$$\Psi = \tfrac{1}{2} - \tfrac{\alpha-1}{u}$$

$$E = p - \int_0^{y_0} \phi(u)du$$

$$\phi(u) = \tfrac{1}{2^{\alpha}\Gamma(\alpha)} e^{-u/2} u^{\alpha-1}.$$

For most values of p and α the starting value

$$y_{01} = 2\alpha \left(z\sqrt{\frac{1}{9\alpha}} + 1 - \frac{1}{9\alpha}\right)^3$$

is used, where z is the deviate associated with a lower tail probability of p for the standard Normal distribution.

For p close to zero,

$$y_{02} = \left(p\alpha 2^{\alpha}\Gamma\left(\alpha\right)\right)^{1/\alpha}$$

is used.

For large p values, when $y_{01} > 4.4\alpha + 6.0$

$$y_{03} = -2\left(\ln(1-p) - (\alpha-1)\ln(\frac{1}{2}y_{01}) + \ln\left(\Gamma\left(\alpha\right)\right)\right)$$

is found to be a better starting value than y_{01}.

For small α $(\alpha \leq 0.16)p$ is expressed in terms of an approximation to the exponential integral and y_{04} is found by Newton–Raphson iterations.

Seven terms of the Taylor series are used to refine the starting approximation, repeating the process if necessary until the required accuracy is obtained.

4 References

[1] Best D J and Roberts D E (1975) Algorithm AS91. The percentage points of the χ^2 distribution *Appl. Statist.* **24** 385–388

5 Parameters

1: P — ***real*** *Input*

On entry: the probability, p, from the required gamma distribution.

Constraint: $0.0 \leq \mathrm{P} < 1.0$.

2: A — ***real*** *Input*

On entry: the shape parameter, α, of the gamma distribution.

Constraint: $0.0 < \mathrm{A} \leq 10^6$.

3: B — ***real*** *Input*

On entry: the scale parameter, β, of the gamma distribution.

Constraint: $\mathrm{B} > 0$.

4: TOL — ***real*** *Input*

On entry: the relative accuracy required by the user in the results. The smallest recommended value is $50 \times \delta$, where $\delta = \max(10^{-18}$, ***machine precision***). If G01FFF is entered with TOL less than $10 \times \delta$ or greater or equal to 1.0, then $10 \times \delta$ is used instead.

5: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error or gives a warning (see Section 6).

For this routine, because the values of output parameters may be useful even if IFAIL $\neq$ 0 on exit, users are recommended to set IFAIL to -1 before entry. **It is then essential to test the value of IFAIL on exit**.

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings specified by the routine:

If on exit IFAIL = 1, 2, 3 or 5, then G01FFF returns 0.0.

IFAIL = 1

On entry, $\mathrm{P} < 0.0$,
or $\mathrm{P} \geq 1.0$,

IFAIL = 2

On entry, A $\leq$ 0.0,
or A $> 10^6$,
or B $\leq$ 0.0

IFAIL = 3

P is too close to 0.0 or 1.0 to enable the result to be calculated.

IFAIL = 4

The solution has failed to converge in 100 iterations. A larger value of TOL should be tried. The result may be a reasonable approximation.

IFAIL = 5

The series to calculate the gamma probabilities has failed to converge. This is an unlikely error exit.

7 Accuracy

In most cases the relative accuracy of the results should be as specified by TOL. However for very small values of α or very small values of p there may be some loss of accuracy.

8 Further Comments

None.

9 Example

Lower tail probabilities are read for several gamma distributions, and the corresponding deviates calculated and printed, until the end of data is reached.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01FFF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             A, B, P, X
      INTEGER          IFAIL
*     .. External Functions ..
      real             G01FFF
      EXTERNAL         G01FFF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01FFF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '   P       A       B           X'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) P, A, B
      IFAIL = -1
```

```
*
      X = G01FFF(P,A,B,0.0e0,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) P, A, B, X
      ELSE
         WRITE (NOUT,99999) P, A, B, X, ' NOTE: IFAIL = ', IFAIL
      END IF
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,3F8.3,F10.3,A,I1)
      END
```

9.2 Program Data

```
G01FFF Example Program Data.
 0.0100   1.0 20.0         :P A B
 0.4279   7.5  0.1         :P A B
 0.8694  45.0 10.0         :P A B
```

9.3 Program Results

```
 G01FFF Example Program Results

    P       A       B           X

    0.010   1.000  20.000     0.201
    0.428   7.500   0.100     0.670
    0.869  45.000  10.000   525.979
```
