NAG Fortran Library Routine Document

G08AJF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G08AJF calculates the exact tail probability for the Mann–Whitney rank sum test statistic for the case where there are no ties in the two samples pooled together.

2 Specification

```
SUBROUTINE G08AJF(N1, N2, TAIL, U, P, WRK, LWRK, IFAIL)
INTEGER            N1, N2, LWRK, IFAIL
real               U, P, WRK(LWRK)
CHARACTER*1        TAIL
```

3 Description

G08AJF computes the exact tail probability for the Mann–Whitney U test statistic (calculated by G08AHF and returned through the parameter U) using a method based on an algorithm developed by Harding (1983), and presented by Neumann (1988), for the case where there are no ties in the pooled sample.

The Mann–Whitney U test investigates the difference between two populations defined by the distribution functions $F(x)$ and $G(y)$ respectively. The data consist of two independent samples of size n_1 and n_2, denoted by $x_1, x_2, \ldots, x_{n_1}$ and $y_1, y_2, \ldots, y_{n_2}$, taken from the two populations.

The hypothesis under test, H_0, often called the null hypothesis, is that the two distributions are the same, that is $F(x) = G(x)$, and this is to be tested against an alternative hypothesis H_1 which is

$H_1 : F(x) \neq G(y)$; or

$H_1 : F(x) < G(y)$, i.e., the x's tend to be greater than the y's; or

$H_1 : F(x) > G(y)$, i.e., the x's tend to be less than the y's,

using a two-tailed, upper-tailed or lower-tailed probability respectively. The user selects the alternative hypothesis by choosing the appropriate tail probability to be computed (see the description of argument TAIL in Section 5).

Note that when using this test to test for differences in the distributions one is primarily detecting differences in the location of the two distributions. That is to say, if we reject the null hypothesis H_0 in favour of the alternative hypothesis H_1: $F(x) > G(y)$ we have evidence to suggest that the location, of the distribution defined by $F(x)$, is less than the location, of the distribution defined by $G(y)$.

G08AJF returns the exact tail probability, p, corresponding to U, depending on the choice of alternative hypothesis, H_1.

The value of p can be used to perform a significance test on the null hypothesis H_0 against the alternative hypothesis H_1. Let α be the size of the significance test (that is, α is the probability of rejecting H_0 when H_0 is true). If $p < \alpha$ then the null hypothesis is rejected. Typically α might be 0.05 or 0.01.

4 References

Conover W J (1980) *Practical Nonparametric Statistics* Wiley

Harding E F (1983) An efficient minimal-storage procedure for calculating the Mann–Whitney U, generalised U and similar distributions *Appl. Statist.* **33** 1–6

Neumann N (1988) Some procedures for calculating the distributions of elementary nonparametric teststatistics *Statistical Software Newsletter* **14 (3)** 120–126

Siegel S (1956) *Non-parametric Statistics for the Behavioral Sciences* McGraw-Hill

5 Parameters

1: N1 – INTEGER *Input*

On entry: the number of non-tied pairs, n_1.

Constraint: N1 $\geq$ 1.

2: N2 – INTEGER *Input*

On entry: the size of the second sample, n_2.

Constraint: N2 $\geq$ 1.

3: TAIL – CHARACTER*1 *Input*

On entry: indicates the choice of tail probability, and hence the alternative hypothesis.

If TAIL = 'T', then a two-tailed probability is calculated and the alternative hypothesis is $H_1 : F(x) \neq G(y)$.

If TAIL = 'U', then an upper-tailed probability is calculated and the alternative hypothesis $H_1 : F(x) < G(y)$, i.e., the x's tend to be greater than the y's.

If TAIL = 'L', then a lower-tailed probability is calculated and the alternative hypothesis $H_1 : F(x) > G(y)$, i.e., the x's tend to be less than the y's.

Constraint: TAIL = 'T', 'U' or 'L'.

4: U – ***real*** *Input*

On entry: the value of the Mann–Whitney rank sum test statistic, U. This is the statistic returned through the parameter U by G08AHF.

Constraint: U $\geq$ 0.0.

5: P – ***real*** *Output*

On exit: the exact tail probability, p, as specified by the parameter TAIL.

6: WRK(LWRK) – ***real*** array *Workspace*

7: LWRK – INTEGER *Input*

On entry: the dimension of the array WRK as declared in the (sub)program from which G08AJF is called.

Constraint: LWRK $\geq$ (N1 $\times$ N2)$/2 + 1$.

8: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, N1 < 1,
or N2 < 1.

IFAIL = 2

On entry, TAIL $\neq$ 'T', 'U' or 'L'.

IFAIL = 3

On entry, U < 0.0.

IFAIL = 4

On entry, LWRK < $(\mathrm{N1} \times \mathrm{N2})/2 + 1$.

7 Accuracy

The exact tail probability, p, is computed to an accuracy of at least 4 significant figures.

8 Further Comments

The time taken by the routine increases with n_1 and n_2 and the product $n_1 n_2$.

9 Example

The example program finds the Mann–Whitney test statistic, using G08AHF for two independent samples of size 16 and 23 respectively. This is used to test the null hypothesis that the distributions of the two populations from which the samples were taken are the same against the alternative hypothesis that the distributions are different. The test statistic, the approximate normal statistic and the approximate two-tail probability are printed. G08AJF is then called to obtain the exact two-tailed probability. The exact probability is also printed.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G08AJF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MAXN1, MAXN2, MAXL
      PARAMETER        (MAXN1=25,MAXN2=25,MAXL=200)
*     .. Local Scalars ..
      real             P, PEXACT, U, UNOR
      INTEGER          I, IFAIL, LWRK, N1, N2
      LOGICAL          TIES
*     .. Local Arrays ..
      real             RANKS(MAXN1+MAXN2), WRK(MAXL), X(MAXN1), Y(MAXN2)
*     .. External Subroutines ..
      EXTERNAL         G08AHF, G08AJF
*     .. Intrinsic Functions ..
      INTRINSIC        INT
*     .. Executable Statements ..
```

```
      WRITE (NOUT,*) 'G08AJF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N1, N2
      WRITE (NOUT,*)
      IF (N1.LE.MAXN1 .AND. N2.LE.MAXN2) THEN
         WRITE (NOUT,99999) 'Sample size of group 1 = ', N1
         WRITE (NOUT,99999) 'Sample size of group 2 = ', N2
         WRITE (NOUT,*)
         READ (NIN,*) (X(I),I=1,N1)
         WRITE (NOUT,*) 'Mann-Whitney U test'
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'Data values'
         WRITE (NOUT,*)
         WRITE (NOUT,99998) '    Group 1  ', (X(I),I=1,N1)
         READ (NIN,*) (Y(I),I=1,N2)
         WRITE (NOUT,*)
         WRITE (NOUT,99998) '    Group 2  ', (Y(I),I=1,N2)
         IFAIL = 0
*
         CALL G08AHF(N1,X,N2,Y,'Lower-tail',U,UNOR,P,TIES,RANKS,WRK,
     +               IFAIL)
*
         WRITE (NOUT,*)
         WRITE (NOUT,99997) 'Test statistic     = ', U
         WRITE (NOUT,99997) 'Normal statistic   = ', UNOR
         WRITE (NOUT,99997) 'Tail probability   = ', P
         WRITE (NOUT,*)
         IF ( .NOT. TIES) THEN
            LWRK = INT(N1*N2/2) + 1
            WRITE (NOUT,99996)
     +        'The length of the workspace is calculated as ', LWRK
            IFAIL = 0
*
            CALL G08AJF(N1,N2,'Lower-tail',U,PEXACT,WRK,LWRK,IFAIL)
*
            WRITE (NOUT,*)
            WRITE (NOUT,99997) 'Exact tail probability = ', PEXACT
         ELSE
            WRITE (NOUT,*)
     +   'There are ties in the pooled sample so G08AJF was not called.'
         END IF
      ELSE
         WRITE (NOUT,*) 'Either N1 or N2 is out of range :'
         WRITE (NOUT,99995) 'N1 = ', N1, ' and N2 = ', N2
      END IF
      STOP
*
99999 FORMAT (1X,A,I5)
99998 FORMAT (1X,A,8F5.1,2(/14X,8F5.1))
99997 FORMAT (1X,A,F10.4)
99996 FORMAT (1X,A,I10)
99995 FORMAT (1X,A,I16,A,I16)
      END
```

9.2 Program Data

```
G08AJF Example Program Data
 16 23
 13.0  5.8 11.7  6.5 12.3  6.7  9.2  6.9
 10.0  7.3 16.0  7.0 10.5  8.5  9.0  7.5
 17.0  6.2 10.1  8.0 15.3  8.2 15.0  9.6 14.9 10.4 14.2  9.8
 13.8 11.0 14.0 11.1 12.9 11.6 12.8 12.0 13.1 12.4 11.9
```

9.3 Program Results

```
G08AJF Example Program Results

Sample size of group 1 =    16
Sample size of group 2 =    23

Mann-Whitney U test

Data values

    Group 1   13.0  5.8 11.7  6.5 12.3  6.7  9.2  6.9
              10.0  7.3 16.0  7.0 10.5  8.5  9.0  7.5


    Group 2   17.0  6.2 10.1  8.0 15.3  8.2 15.0  9.6
              14.9 10.4 14.2  9.8 13.8 11.0 14.0 11.1
              12.9 11.6 12.8 12.0 13.1 12.4 11.9

Test statistic     =    86.0000
Normal statistic   =    -2.7838
Tail probability   =     0.0027

The length of the workspace is calculated as        185

Exact tail probability =     0.0022
```