

G13CAF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

G13CAF calculates the smoothed sample spectrum of a univariate time series using one of four lag windows – rectangular, Bartlett, Tukey or Parzen window.

2 Specification

```

SUBROUTINE G13CAF(NX, MTX, PX, IW, MW, IC, NC, C, KC, L, LG, NXG,
1              XG, NG, STATS, IFAIL)
  INTEGER      NX, MTX, IW, MW, IC, NC, KC, L, LG, NXG, NG,
1              IFAIL
  real         PX, C(NC), XG(NXG), STATS(4)

```

3 Description

The smoothed sample spectrum is defined as

$$\hat{f}(\omega) = \frac{1}{2\pi} \left(C_0 + 2 \sum_{k=1}^{M-1} w_k C_k \cos(\omega k) \right)$$

where M is the window width, and is calculated for frequency values

$$\omega_i = \frac{2\pi i}{L}, \quad i = 0, 1, \dots, [L/2]$$

where $[]$ denotes the integer part.

The autocovariances C_k may be supplied by the user, or constructed from a time series x_1, x_2, \dots, x_n , as

$$C_k = \frac{1}{n} \sum_{t=1}^{n-k} x_t x_{t+k}$$

the fast Fourier transform (FFT) being used to carry out the convolution in this formula.

The time series may be mean or trend corrected (by classical least squares), and tapered before calculation of the covariances, the tapering factors being those of the split cosine bell:

$$\begin{aligned} & \frac{1}{2} \left(1 - \cos\left(\pi\left(t - \frac{1}{2}\right)/T\right) \right), & 1 \leq t \leq T \\ & \frac{1}{2} \left(1 - \cos\left(\pi\left(n - t + \frac{1}{2}\right)/T\right) \right), & n + 1 - T \leq t \leq n \\ & 1, & \text{otherwise} \end{aligned}$$

where $T = \left[\frac{np}{2} \right]$ and p is the tapering proportion.

The smoothing window is defined by

$$w_k = W\left(\frac{k}{M}\right), \quad k \leq M - 1$$

which for the various windows is defined over $0 \leq \alpha < 1$ by

rectangular:

$$W(\alpha) = 1$$

Bartlett:

$$W(\alpha) = 1 - \alpha$$

Tukey:

$$W(\alpha) = \frac{1}{2}(1 + \cos(\pi\alpha))$$

Parzen:

$$W(\alpha) = 1 - 6\alpha^2 + 6\alpha^3, \quad 0 \leq \alpha \leq \frac{1}{2}$$

$$W(\alpha) = 2(1 - \alpha)^3, \quad \frac{1}{2} < \alpha < 1.$$

The sampling distribution of $\hat{f}(\omega)$ is approximately that of a scaled χ_d^2 variate, whose degrees of freedom d is provided by the routine, together with multiplying limits mu , ml from which approximate 95% confidence intervals for the true spectrum $f(\omega)$ may be constructed as $[ml \times \hat{f}(\omega), mu \times \hat{f}(\omega)]$. Alternatively, $\log \hat{f}(\omega)$ may be returned, with additive limits.

The bandwidth b of the corresponding smoothing window in the frequency domain is also provided. Spectrum estimates separated by (angular) frequencies much greater than b may be assumed to be independent.

4 References

- [1] Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day
- [2] Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

- 1: NX — INTEGER *Input*
On entry: the length of the time series, n .
Constraint: $NX \geq 1$.

- 2: MTX — INTEGER *Input*
On entry: if covariances are to be calculated by the routine ($IC = 0$), MTX must specify whether the data are to be initially mean or trend corrected.

MTX = 0 for no correction,
MTX = 1 for mean correction,
MTX = 2 for trend correction.
Constraint: $0 \leq MTX \leq 2$, if $IC = 0$.

If covariances are supplied ($IC \neq 0$), MTX is not used.

- 3: PX — *real* *Input*
On entry: if covariances are to be calculated by the routine ($IC = 0$), PX must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. If covariances are supplied ($IC \neq 0$), then PX must specify the proportion of data tapered before the supplied covariances were calculated and after any mean or trend correction. PX is required for the calculation of output statistics. A value of 0.0 implies no tapering.

Constraint: $0.0 \leq PX \leq 1.0$.

- 4: IW — INTEGER *Input*
On entry: the choice of lag window. IW = 1 for rectangular, 2 for Bartlett, 3 for Tukey or 4 for Parzen.

Constraint: $1 \leq IW \leq 4$.

- 5:** MW — INTEGER *Input*
On entry: the ‘cut-off’ point, M , of the lag window. Windowed covariances at lag M or greater are zero.
Constraint: $1 \leq MW \leq NX$.
- 6:** IC — INTEGER *Input*
On entry: indicates whether covariances are to be calculated in the routine or supplied in the call to the routine.
 IC = 0 if covariances are to be calculated,
 IC \neq 0 if covariances are to be supplied.
- 7:** NC — INTEGER *Input*
On entry: the number of covariances to be calculated in the routine or supplied in the call to the routine.
Constraint: $MW \leq NC \leq NX$.
- 8:** C(NC) — *real* array *Input/Output*
On entry: if IC \neq 0, then C must contain the NC covariances for lags from 0 to (NC – 1), otherwise C need not be set.
On exit: if IC = 0, C will contain the NC calculated covariances.
 If IC \neq 0, the contents of C will be unchanged.
- 9:** KC — INTEGER *Input*
On entry: if IC = 0, KC must specify the order of the fast Fourier transform (FFT) used to calculate the covariances. KC should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq NX + NC$, provided $m \leq 20$.
 If IC \neq 0, that is covariances are supplied, then KC is not used.
Constraints:
 $KC \geq NX + NC$.
 The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF which performs the FFT.
- 10:** L — INTEGER *Input*
On entry: the frequency division, L , of the spectral estimates as $\frac{2\pi}{L}$. Therefore it is also the order of the FFT used to construct the sample spectrum from the covariances. L should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2M - 1$, provided $m \leq 20$.
Constraints:
 $L \geq 2 \times MW - 1$.
 The largest prime factor of L must not exceed 19, and the total number of prime factors of L, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF which performs the FFT.
- 11:** LG — INTEGER *Input*
On entry: indicates whether unlogged or logged spectral estimates and confidence limits are required.
 LG = 0 for unlogged, LG \neq 0 for logged.

- 12: NXG — INTEGER** *Input*
On entry: the length of the array XG, as declared in the (sub)program from which G13CAF is called.
Constraints:
 if covariances are to be calculated, i.e., IC = 0, $NXG \geq \max(KC, L)$,
 if covariances are supplied, i.e., if IC \neq 0, $NXG \geq L$.
- 13: XG(NXG) — *real* array** *Input/Output*
On entry: if the covariances are to be calculated, then XG must contain the NX data points. If covariances are supplied, XG may contain any values.
On exit: contains the NG spectral estimates, $\hat{f}(\omega_i)$, for $i = 0, 1, \dots, [L/2]$ in XG(1) to XG(NG) respectively (logged if LG \neq 0). The elements XG(i), for $i = NG + 1, \dots, NXG$ contain 0.0.
- 14: NG — INTEGER** *Output*
On exit: the number of spectral estimates, $[L/2] + 1$, in XG.
- 15: STATS(4) — *real* array** *Output*
On exit: four associated statistics. These are the degrees of freedom in STATS(1), the lower and upper 95% confidence limit factors in STATS(2) and STATS(3) respectively (logged if LG \neq 0), and the bandwidth in STATS(4).
- 16: IFAIL — INTEGER** *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

- On entry, $NX < 1$,
- or $MTX < 0$ and $IC = 0$,
- or $MTX > 2$ and $IC = 0$,
- or $PX < 0.0$,
- or $PX > 1.0$,
- or $IW < 1$,
- or $IW > 4$,
- or $MW < 1$,
- or $MW > NX$,
- or $NC < MW$,
- or $NC > NX$,
- or $NXG < \max(KC, L)$ and $IC = 0$,
- or $NXG < L$ and $IC \neq 0$.

IFAIL = 2

- On entry, $KC < NX + NC$,
- or KC has a prime factor exceeding 19,
- or KC has more than 20 prime factors, counting repetitions.

This error only occurs when $IC = 0$.

IFAIL = 3

On entry, $L < 2 \times MW - 1$,
 or L has a prime factor exceeding 19,
 or L has more than 20 prime factors, counting repetitions.

IFAIL = 4

One or more spectral estimates are negative. Unlogged spectral estimates are returned in XG, and the degrees of freedom, unlogged confidence limit factors and bandwidth in STATS.

IFAIL = 5

The calculation of confidence limit factors has failed. This error will not normally occur. Spectral estimates (logged if requested) are returned in XG, and degrees of freedom and bandwidth in STATS.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

G13CAF carries out two FFTs of length KC to calculate the covariances and one FFT of length L to calculate the sample spectrum. The time taken by the routine for an FFT of length n is approximately proportional to $n \log(n)$ (see Section 8 of the document for C06EAF for further details).

9 Example

The example program reads a time series of length 256. It selects the mean correction option, a tapering proportion of 0.1, the Parzen smoothing window and a cut-off point for the window at lag 100. It chooses to have 100 auto-covariances calculated and unlogged spectral estimates at a frequency division of $2\pi/200$. It then calls G13CAF to calculate the univariate spectrum and statistics and prints the autocovariances and the spectrum together with its 95% confidence multiplying limits.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      G13CAF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NXG, NCMAX
      PARAMETER        (NXG=500,NCMAX=200)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real            PX
      INTEGER          I, IC, IFAIL, IW, KC, L, LG, MTX, MW, NC, NG, NX
*      .. Local Arrays ..
      real            C(NCMAX), STATS(4), XG(NXG)
*      .. External Subroutines ..
      EXTERNAL         G13CAF
*      .. Executable Statements ..
```

```

WRITE (NOUT,*) 'G13CAF Example Program Results'
*
Skip heading in data file
READ (NIN,*)
READ (NIN,*) NX, NC
IF (NX.GT.0 .AND. NX.LE.NXG .AND. NC.GT.0 .AND. NC.LE.NCMAX) THEN
  READ (NIN,*) (XG(I),I=1,NX)
  MTX = 1
  PX = 0.1e0
  IW = 4
  MW = 100
  IC = 0
  KC = 360
  L = 200
  LG = 0
  IFAIL = 1
*
  CALL G13CAF(NX,MTX,PX,IW,MW,IC,NC,C,KC,L,LG,NXG,XG,NG,STATS,
+          IFAIL)
*
  WRITE (NOUT,*)
  IF (IFAIL.NE.0) THEN
    WRITE (NOUT,99999) 'G13CAF fails. IFAIL =', IFAIL
    WRITE (NOUT,*)
  END IF
  IF (IFAIL.EQ.0 .OR. IFAIL.GE.4) THEN
    WRITE (NOUT,*) 'Covariances'
    WRITE (NOUT,99998) (C(I),I=1,NC)
    WRITE (NOUT,*)
    WRITE (NOUT,99997) 'Degrees of freedom =', STATS(1),
+      ' Bandwidth =', STATS(4)
    WRITE (NOUT,*)
    WRITE (NOUT,99996)
+      '95 percent confidence limits - Lower =', STATS(2),
+      ' Upper =', STATS(3)
    WRITE (NOUT,*)
    WRITE (NOUT,*)
+      ' Spectrum Spectrum Spectrum Spectrum'
    WRITE (NOUT,*)
+      ' estimate estimate estimate estimate'
    WRITE (NOUT,99995) (I,XG(I),I=1,NG)
  END IF
  END IF
  STOP
*
99999 FORMAT (1X,A,I3)
99998 FORMAT (1X,6F11.4)
99997 FORMAT (1X,A,F4.1,A,F7.4)
99996 FORMAT (1X,A,F7.4,A,F7.4)
99995 FORMAT (1X,I4,F10.4,I5,F10.4,I5,F10.4,I5,F10.4)
END

```

9.2 Program Data

G13CAF Example Program Data

256 100

5.0	11.0	16.0	23.0	36.0	58.0	29.0	20.0	10.0	8.0	3.0	0.0
0.0	2.0	11.0	27.0	47.0	63.0	60.0	39.0	28.0	26.0	22.0	11.0
21.0	40.0	78.0	122.0	103.0	73.0	47.0	35.0	11.0	5.0	16.0	34.0
70.0	81.0	111.0	101.0	73.0	40.0	20.0	16.0	5.0	11.0	22.0	40.0
60.0	80.9	83.4	47.7	47.8	30.7	12.2	9.6	10.2	32.4	47.6	54.0
62.9	85.9	61.2	45.1	36.4	20.9	11.4	37.8	69.8	106.1	100.8	81.6
66.5	34.8	30.6	7.0	19.8	92.5	154.4	125.9	84.8	68.1	38.5	22.8
10.2	24.1	82.9	132.0	130.9	118.1	89.9	66.6	60.0	46.9	41.0	21.3
16.0	6.4	4.1	6.8	14.5	34.0	45.0	43.1	47.5	42.2	28.1	10.1
8.1	2.5	0.0	1.4	5.0	12.2	13.9	35.4	45.8	41.1	30.1	23.9
15.6	6.6	4.0	1.8	8.5	16.6	36.3	49.6	64.2	67.0	70.9	47.8
27.5	8.5	13.2	56.9	121.5	138.3	103.2	85.7	64.6	36.7	24.2	10.7
15.0	40.1	61.5	98.5	124.7	96.3	66.6	64.5	54.1	39.0	20.6	6.7
4.3	22.7	54.8	93.8	95.8	77.2	59.1	44.0	47.0	30.5	16.3	7.3
37.6	74.0	139.0	111.2	101.6	66.2	44.7	17.0	11.3	12.4	3.4	6.0
32.3	54.3	59.7	63.7	63.5	52.2	25.4	13.1	6.8	6.3	7.1	35.6
73.0	85.1	78.0	64.0	41.8	26.2	26.7	12.1	9.5	2.7	5.0	24.4
42.0	63.5	53.8	62.0	48.5	43.9	18.6	5.7	3.6	1.4	9.6	47.4
57.1	103.9	80.6	63.6	37.6	26.1	14.2	5.8	16.7	44.3	63.9	69.0
77.8	64.9	35.7	21.2	11.1	5.7	8.7	36.1	79.7	114.4	109.6	88.8
67.8	47.5	30.6	16.3	9.6	33.2	92.6	151.6	136.3	134.7	83.9	69.4
31.5	13.9	4.4	38.0								

9.3 Program Results

G13CAF Example Program Results

Covariances

1152.9733	937.3289	494.9243	14.8648	-342.8548	-514.6479
-469.2733	-236.6896	109.0608	441.3498	637.4571	641.9954
454.0505	154.5960	-136.8016	-343.3911	-421.8441	-374.4095
-241.1943	-55.6140	129.4067	267.4248	311.8293	230.2807
56.4402	-146.4689	-320.9948	-406.4077	-375.6384	-273.5936
-132.6214	11.0791	126.4843	171.3391	122.6284	-11.5482
-169.2623	-285.2358	-331.4567	-302.2945	-215.4832	-107.8732
-3.4126	73.2521	98.0831	71.8949	17.0985	-27.5632
-76.7900	-110.5354	-126.1383	-121.1043	-103.9362	-67.4619
-10.8678	58.5009	116.4587	140.0961	129.5928	66.3211
-35.5487	-135.3894	-203.7149	-216.2161	-152.7723	-30.4361
99.3397	188.9594	204.9047	148.4056	34.4975	-103.7840
-208.5982	-252.4128	-223.7600	-120.8640	23.3565	156.0956
227.7642	228.5123	172.3820	87.4911	-21.2170	-117.5282
-176.3634	-165.1218	-75.1308	67.1634	195.7290	279.3039
290.8258	225.3811	104.0784	-44.4731	-162.7355	-207.7480
-165.2444	-48.5473	118.8872	265.0045		

Degrees of freedom = 9.0

Bandwidth = 0.1165

95 percent confidence limits -

Lower = 0.4731 Upper = 3.3329

	Spectrum estimate		Spectrum estimate		Spectrum estimate		Spectrum estimate
1	210.4696	2	428.2020	3	810.1419	4	922.5900
5	706.1605	6	393.4052	7	207.6481	8	179.0657

9	170.1320	10	133.0442	11	103.6752	12	103.0644
13	141.5173	14	194.3041	15	266.5730	16	437.0181
17	985.3130	18	2023.1574	19	2681.8980	20	2363.7439
21	1669.9001	22	1012.1320	23	561.4822	24	467.2741
25	441.9977	26	300.1985	27	172.0184	28	114.7823
29	79.1533	30	49.4882	31	27.0902	32	16.8081
33	27.5111	34	59.4429	35	97.0145	36	119.3664
37	116.6737	38	87.3142	39	54.9570	40	42.9781
41	46.6097	42	53.6206	43	50.6050	44	36.7780
45	25.6285	46	24.8555	47	30.2626	48	31.5642
49	27.3351	50	22.4443	51	18.5418	52	15.2425
53	12.0207	54	12.6846	55	18.3975	56	19.3058
57	12.6103	58	7.9511	59	7.1333	60	5.4996
61	3.4182	62	3.2359	63	5.3836	64	8.5225
65	10.0610	66	7.9483	67	4.2261	68	3.2631
69	5.5751	70	7.8491	71	9.3694	72	11.0791
73	10.1386	74	6.3158	75	3.6375	76	2.6561
77	1.8026	78	1.0103	79	1.0693	80	2.3950
81	4.0822	82	4.6221	83	4.0672	84	3.8460
85	4.8489	86	6.3964	87	6.4762	88	4.9457
89	4.4444	90	5.2131	91	5.0389	92	4.6141
93	5.8722	94	7.9268	95	7.9486	96	5.7854
97	4.5495	98	5.2696	99	6.3893	100	6.5216
101	6.2129						
