

G13CFF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

For a bivariate time series, G13CFF calculates the gain and phase together with lower and upper bounds from the univariate and bivariate spectra.

2 Specification

```

SUBROUTINE G13CFF(XG, YG, XYRG, XYIG, NG, STATS, GN, GNLW, GNUP,
1              PH, PHLW, PHUP, IFAIL)
  INTEGER      NG, IFAIL
  real        XG(NG), YG(NG), XYRG(NG), XYIG(NG), STATS(4),
1              GN(NG), GNLW(NG), GNUP(NG), PH(NG), PHLW(NG),
2              PHUP(NG)

```

3 Description

Estimates of the gain $G(\omega)$ and phase $\phi(\omega)$ of the dependency of series y on series x at frequency ω are given by

$$\hat{G}(\omega) = \frac{A(\omega)}{f_{xx}(\omega)}$$

$$\hat{\phi}(\omega) = \cos^{-1} \left(\frac{cf(\omega)}{A(\omega)} \right), \quad \text{if } cf(\omega) \geq 0$$

$$\hat{\phi}(\omega) = 2\pi - \cos^{-1} \left(\frac{cf(\omega)}{A(\omega)} \right), \quad \text{if } cf(\omega) < 0.$$

The quantities used in these definitions are obtained as in Section 3 of the document for G13CEF.

Confidence limits are returned for both gain and phase, but should again be taken as very approximate when the coherency $W(\omega)$, as calculated by G13CEF, is not significant. These are based on the assumption that both $(\hat{G}(\omega)/G(\omega)) - 1$ and $\hat{\phi}(\omega)$ are Normal with variance

$$\frac{1}{d} \left(\frac{1}{W(\omega)} - 1 \right).$$

Although the estimate of $\phi(\omega)$ is always given in the range $[0, 2\pi]$, no attempt is made to restrict its confidence limits to this range.

4 References

- [1] Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day
- [2] Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

- 1: XG(NG) — *real* array *Input*
On entry: the NG univariate spectral estimates, $f_{xx}(\omega)$, for the x series.
- 2: YG(NG) — *real* array *Input*
On entry: the NG univariate spectral estimates, $f_{yy}(\omega)$, for the y series.

- 3:** XYRG(NG) — *real* array *Input*
On entry: the real parts, $cf(\omega)$ of the NG bivariate spectral estimates for the x and y series. The x series leads the y series.
- 4:** XYIG(NG) — *real* array *Input*
On entry: the imaginary parts, $qf(\omega)$, of the NG bivariate spectral estimates for the x and y series. The x series leads the y series.
- Note.** the two univariate and the bivariate spectra must each have been calculated using the same method of smoothing. For rectangular, Bartlett, Tukey or Parzen smoothing windows, the same cut-off point of lag window and the same frequency division of the spectral estimates must be used. For the trapezium frequency smoothing window, the frequency width and the shape of the window and the frequency division of the spectral estimates must be the same. The spectral estimates and statistics must also be unlogged.
- 5:** NG — INTEGER *Input*
On entry: the number of spectral estimates in each of the arrays XG, YG, XYRG and XYIG. It is also the number of gain and phase estimates.
Constraint: $NG \geq 1$.
- 6:** STATS(4) — *real* array *Input*
On entry: the four associated statistics for the univariate spectral estimates for the x and y series. STATS(1) contains the degrees of freedom, STATS(2) and STATS(3) contain the lower and upper bound multiplying factors respectively and STATS(4) holds the bandwidth.
Constraint: $STATS(1) \geq 3.0$.
- 7:** GN(NG) — *real* array *Output*
On exit: the NG gain estimates, $\hat{G}(\omega)$, at each frequency ω .
- 8:** GNLW(NG) — *real* array *Output*
On exit: the NG lower bounds for the NG gain estimates.
- 9:** GNUP(NG) — *real* array *Output*
On exit: the NG upper bounds for the NG gain estimates.
- 10:** PH(NG) — *real* array *Output*
On exit: the NG phase estimates, $\hat{\phi}(\omega)$, at each frequency ω .
- 11:** PHLW(NG) — *real* array *Output*
On exit: the NG lower bounds for the NG phase estimates.
- 12:** PHUP(NG) — *real* array *Output*
On exit: the NG upper bounds for the NG phase estimates.
- 13:** IFAIL — INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

On entry, $NG < 1$,
or $STATS(1) < 3.0$.

IFAIL = 2

A bivariate spectral estimate is zero. For this frequency the gain and the phase and their bounds are set to zero.

IFAIL = 3

A univariate spectral estimate is negative. For this frequency the gain and the phase and their bounds are set to zero.

IFAIL = 4

A univariate spectral estimate is zero. For this frequency the gain and the phase and their bounds are set to zero.

IFAIL = 5

A calculated value of the squared coherency exceeds 1.0. For this frequency the squared coherency is reset to 1.0 in the formulae for the gain and phase bounds.

If more than one failure of types 2, 3, 4 and 5 occurs then the failure type which occurred at lowest frequency is returned in IFAIL. However the actions indicated above are also carried out for failures at higher frequencies.

7 Accuracy

All computations are very stable and yield good accuracy.

8 Further Comments

The time taken by the routine is approximately proportional to NG .

9 Example

The example program reads the set of univariate spectrum statistics, the 2 univariate spectra and the cross spectrum at a frequency division of $\frac{2\pi}{20}$ for a pair of time series. It calls G13CFF to calculate the gain and the phase and their bounds and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      G13CFF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NGMAX
      PARAMETER        (NGMAX=9)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, J, NG
```

```

*   .. Local Arrays ..
  real      GN(NGMAX), GNLW(NGMAX), GNUP(NGMAX), PH(NGMAX),
+          PHLW(NGMAX), PHUP(NGMAX), STATS(4), XG(NGMAX),
+          XYIG(NGMAX), XYRG(NGMAX), YG(NGMAX)
*   .. External Subroutines ..
  EXTERNAL  G13CFF
*   .. Executable Statements ..
  WRITE (NOUT,*) 'G13CFF Example Program Results'
*   Skip heading in data file
  READ (NIN,*)
  READ (NIN,*) NG
  IF (NG.GT.0 .AND. NG.LE.NGMAX) THEN
    READ (NIN,*) (STATS(I),I=1,4)
    READ (NIN,*) (XG(I),YG(I),XYRG(I),XYIG(I),I=1,NG)
    IFAIL = 1
*
    CALL G13CFF(XG,YG,XYRG,XYIG,NG,STATS,GN,GNLW,GNUP,PH,PHLW,PHUP,
+             IFAIL)
*
    WRITE (NOUT,*)
    IF (IFAIL.NE.0) THEN
      WRITE (NOUT,99999) 'G13CFF fails. IFAIL =', IFAIL
      WRITE (NOUT,*)
    END IF
    IF (IFAIL.NE.1) THEN
      WRITE (NOUT,*) '          The gain'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          Lower      Upper'
      WRITE (NOUT,*) '          Value      bound      bound'
      DO 20 J = 1, NG
        WRITE (NOUT,99998) J - 1, GN(J), GNLW(J), GNUP(J)
20      CONTINUE
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          The phase'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          Lower      Upper'
      WRITE (NOUT,*) '          Value      bound      bound'
      DO 40 J = 1, NG
        WRITE (NOUT,99998) J - 1, PH(J), PHLW(J), PHUP(J)
40      CONTINUE
      END IF
    END IF
  STOP
*
99999 FORMAT (1X,A,I3)
99998 FORMAT (1X,I5,3F10.4)
  END

```

9.2 Program Data

G13CFF Example Program Data

```

9
30.00000 .63858 1.78670 .33288
2.03490 21.97712 -6.54995 0.00000
.51554 3.29761 .34107 -1.19030
.07640 .28782 .12335 .04087
.01068 .02480 -.00514 .00842
.00093 .00285 -.00033 .00032
.00100 .00203 -.00039 -.00001
.00076 .00125 -.00026 .00018
.00037 .00107 .00011 -.00016
.00021 .00191 .00007 0.00000

```

9.3 Program Results

G13CFF Example Program Results

The gain

	Value	Lower bound	Upper bound
0	3.2188	2.9722	3.4859
1	2.4018	2.1138	2.7290
2	1.7008	1.3748	2.1042
3	0.9237	0.5558	1.5350
4	0.4943	0.1327	1.8415
5	0.3901	0.1002	1.5196
6	0.4161	0.1346	1.2863
7	0.5248	0.1591	1.7306
8	0.3333	0.0103	10.8301

The phase

	Value	Lower bound	Upper bound
0	3.1416	3.0619	3.2213
1	4.9915	4.8637	5.1192
2	0.3199	0.1071	0.5328
3	2.1189	1.6109	2.6268
4	2.3716	1.0563	3.6868
5	3.1672	1.8075	4.5270
6	2.5360	1.4074	3.6647
7	5.3147	4.1214	6.5079
8	0.0000	-3.4809	3.4809
