

# NAG Fortran Library Routine Document

## G13FEF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

G13FEF estimates the parameters of a univariate regression-GJR GARCH( $p, q$ ) process (see Glosten *et al.* (1993)).

### 2 Specification

```

SUBROUTINE G13FEF(DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, NPAR, THETA,
1          SE, SC, COVAR, LDC, HP, ET, HT, LGF, COPTS, MAXIT,
2          TOL, WORK, LWORK, IFAIL)
  INTEGER          LDX, NUM, IP, IQ, NREG, MN, NPAR, LDC, MAXIT, LWORK,
1          IFAIL
  real           YT(NUM), X(LDX,*), THETA(NPAR), SE(NPAR), SC(NPAR),
1          COVAR(LDC,NPAR), HP, ET(NUM), HT(NUM), LGF, TOL,
2          WORK(LWORK)
  LOGICAL         COPTS(2)
  CHARACTER*1     DIST

```

### 3 Description

A univariate regression-GJR GARCH( $p, q$ ) process, with  $q$  coefficients  $\alpha_i$ ,  $i = 1, \dots, q$ ,  $q$  coefficients,  $\beta_i$ , for  $i = 1, \dots, p$  and  $k$  linear regression coefficients  $b_i$ , for  $i = 1, \dots, k$ , can be represented by:

$$y_t = b_o + x_t^T b + \epsilon_t \quad (1)$$

$$h_t = \alpha_0 + \sum_{i=1}^q (\alpha_i + \gamma S_{t-i}) \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}, \quad t = 1, \dots, T \quad (2)$$

where  $S_t = 1$ , if  $\epsilon_t < 0$ ,  $S_t = 0$ , if  $\epsilon_t \geq 0$ , and  $\epsilon_t | \psi_{t-1} = N(0, h_t)$  or  $\epsilon_t | \psi_{t-1} = S_t(df, h_t)$ . Here  $S_t$  is a standardised Student's  $t$ -distribution with  $df$  degrees of freedom and variance  $h_t$ ,  $T$  is the number of terms in the sequence,  $y_t$  denotes the endogenous variables,  $x_t$  the exogenous variables,  $b_o$  the regression mean,  $b$  the regression coefficients,  $\epsilon_t$  the residuals,  $h_t$  is the conditional variance, and  $\psi_t$  the set of all information up to time  $t$ .

G13FEF provides an estimate for  $\hat{\theta}$ , the parameter vector  $\theta = (b_o, b^T, \omega^T)$  where  $b^T = (b_1, \dots, b_k)$ ,  $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma)$  when  $DIST = 'N'$  and  $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma, df)$  when  $DIST = 'T'$ .

MN, NREG can be used to simplify the GARCH( $p, q$ ) expression in (1) as follows:

#### No Regression and No Mean

$$y_t = \epsilon_t,$$

$$MN = 0,$$

$$NREG = 0 \text{ and}$$

$\theta$  is a  $(p + q + 2)$  vector when  $DIST = 'N'$ , and a  $(p + q + 3)$  vector when  $DIST = 'T'$ .

**No Regression**

$$y_t = b_o + \epsilon_t,$$

$$MN = 1,$$

$$NREG = 0 \text{ and}$$

$\theta$  is a  $(p + q + 3)$  vector when DIST = 'N', and a  $(p + q + 4)$  vector when DIST = 'T'.

**Note:** if the  $y_t = \mu + \epsilon_t$ , where  $\mu$  is known (not to be estimated by G13FEF) then (1) can be written as  $y_t^\mu = \epsilon_t$ , where  $y_t^\mu = y_t - \mu$ . This corresponds to the case **No Regression and No Mean**, with  $y_t$  replaced by  $y_t - \mu$ .

**No Mean**

$$y_t = x_t^T b + \epsilon_t,$$

$$MN = 0,$$

$$NREG = k \text{ and}$$

$\theta$  is a  $(p + q + k + 2)$  vector when DIST = 'N', and a  $(p + q + k + 3)$  vector when DIST = 'T'.

**4 References**

Engle R (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation *Econometrica* **50** 987–1008

Bollerslev T (1986) Generalised autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R and Ng V (1993) Measuring and Testing the Impact of News on Volatility *Journal of Finance* **48** 1749–1777

Hamilton J (1994) *Time Series Analysis* Princeton University Press

Glosten L, Jagannathan R and Runkle D (1993) Relationship between the expected value and the volatility of nominal excess return on stocks *Journal of Finance* **48** 1779–1801

**5 Parameters**

- 1: DIST – CHARACTER\*1 *Input*  
*On entry:* the type of distribution to use for  $e_t$ .  
 If DIST = 'N', a Normal distribution is used.  
 If DIST = 'T', a Student's  $t$ -distribution is used.  
*Constraint:* DIST = 'N' or 'T'.
- 2: YT(NUM) – *real* array *Input*  
*On entry:* the sequence of observations,  $y_t$ , for  $t = 1, \dots, T$ .
- 3: X(LDX,\*) – *real* array *Input*  
**Note:** the second dimension of the array X must be at least  $\max(1, NREG + MN)$ .  
*On entry:* row  $t$  of X must contain the time dependent exogenous vector  $x_t$ , where  $x_t^T = (x_t^1, \dots, x_t^k)$ , for  $t = 1, \dots, T$ .
- 4: LDX – INTEGER *Input*  
*On entry:* the first dimension of the array X as declared in the (sub)program from which G13FEF is called.  
*Constraint:* LDX  $\geq$  NUM.

- 5: NUM – INTEGER Input  
*On entry:* the number of terms in the sequence,  $T$ .  
*Constraint:*  $\text{NUM} \geq \max(\text{IP}, \text{IQ})$ .
- 6: IP – INTEGER Input  
*On entry:* the number of coefficients,  $\beta_i$ , for  $i = 1, \dots, p$ .  
*Constraint:*  $\text{IP} \geq 0$  (see also NPAR).
- 7: IQ – INTEGER Input  
*On entry:* the number of coefficients,  $\alpha_i$ , for  $i = 1, \dots, q$ .  
*Constraint:*  $\text{IQ} \geq 1$  (see also NPAR).
- 8: NREG – INTEGER Input  
*On entry:* the number of regression coefficients,  $k$ .  
*Constraint:*  $\text{NREG} \geq 0$  (see also NPAR).
- 9: MN – INTEGER Input  
*On entry:* if  $\text{MN} = 1$ , the mean term  $b_0$  will be included in the model.  
*Constraint:*  $\text{MN} = 0$  or  $1$ .
- 10: NPAR – INTEGER Input  
*On entry:* the number of parameters to be included in the model.  
 $\text{NPAR} = 2 + \text{IQ} + \text{IP} + \text{MN} + \text{NREG}$  when  $\text{DIST} = \text{'N'}$  and  
 $\text{NPAR} = 3 + \text{IQ} + \text{IP} + \text{MN} + \text{NREG}$  when  $\text{DIST} = \text{'T'}$ .  
*Constraint:*  $\text{NPAR} < 20$ .
- 11: THETA(NPAR) – *real* array Input/Output  
*On entry:* the initial parameter estimates for the vector  $\theta$ . The first element must contain the coefficient  $\alpha_o$  and the next IQ elements contain the coefficients  $\alpha_i$ , for  $i = 1, \dots, q$ . The next IP elements must contain the coefficients  $\beta_j$ , for  $j = 1, \dots, p$ . The next element must contain the asymmetry parameter  $\gamma$ . If  $\text{DIST} = \text{'T'}$ , then the next element contains  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution. If  $\text{MN} = 1$ , then the next element must contain the mean term  $b_o$ . If  $\text{COPTS}(2) = \text{.FALSE.}$ , then the remaining NREG elements are taken as initial estimates of the linear regression coefficients  $b_i$ , for  $i = 1, \dots, k$ .  
*On exit:* the estimated values  $\hat{\theta}$  for the vector  $\theta$ . The first element contains the coefficient  $\alpha_o$ , the next IQ elements contain the coefficients  $\alpha_i$ , for  $i = 1, \dots, q$ . The next IP elements are the moving average coefficients  $\beta_j$ , for  $j = 1, \dots, p$ . The next element contains the estimate for the asymmetry parameter  $\gamma$ . If  $\text{DIST} = \text{'T'}$  then the next element contains an estimate for  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution. If  $\text{MN} = 1$  then the next element contains an estimate for the mean term  $b_o$ . The final NREG elements are the estimated linear regression coefficients  $b_i$ , for  $i = 1, \dots, k$ .
- 12: SE(NPAR) – *real* array Output  
*On exit:* the standard errors for  $\hat{\theta}$ . The first element contains the standard error for  $\alpha_o$ , the next IQ elements contain the standard errors for  $\alpha_i$ , for  $i = 1, \dots, q$ , the next IP elements are the standard errors for  $\beta_j$ , for  $j = 1, \dots, p$ . The next element contains the standard error for  $\gamma$ . If  $\text{DIST} = \text{'T'}$  then the next element contains the standard error for  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution. If  $\text{MN} = 1$  then the next element contains the standard error for  $b_o$ . The final NREG elements are the standard errors for  $b_j$ , for  $j = 1, \dots, k$ .

- 13: SC(NPAR) – *real* array *Output*  
*On exit:* the scores for  $\hat{\theta}$ . The first element contains the score for  $\alpha_0$ , the next IQ elements contain the scores for  $\alpha_i$ , for  $i = 1, \dots, q$ , the next IP elements are the score for  $\beta_j$ , for  $j = 1, \dots, p$ . The next element contains the score for  $\gamma$ . If DIST = 'T' then the next element contains the score for  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution. If MN = 1 then the next element contains the score for  $b_0$ . The final NREG elements are the scores for  $b_j$ , for  $j = 1, \dots, k$ .
- 14: COVAR(LDC,NPAR) – *real* array *Output*  
*On exit:* the covariance matrix of the parameter estimates  $\hat{\theta}$ , that is the inverse of the Fisher Information Matrix.
- 15: LDC – INTEGER *Input*  
*On entry:* the first dimension of the array COVAR as declared in the (sub)program from which G13FEF is called.  
*Constraint:* LDC  $\geq$  NPAR.
- 16: HP – *real* *Input/Output*  
*On entry:* if COPTS(2) = .FALSE. then HP is the value to be used for the pre-observed conditional variance, otherwise HP is not referenced.  
*On exit:* if COPTS(2) = .TRUE. then HP is the estimated value of the pre-observed of the conditional variance.
- 17: ET(NUM) – *real* array *Output*  
*On exit:* the estimated residuals,  $\epsilon_t$ , for  $t = 1, \dots, T$ .
- 18: HT(NUM) – *real* array *Output*  
*On exit:* the estimated conditional variances,  $h_t$ , for  $t = 1, \dots, T$ .
- 19: LGF – *real* *Output*  
*On exit:* the value of the log likelihood function at  $\hat{\theta}$ .
- 20: COPTS(2) – LOGICAL array *Input*  
*On entry:* the options to be used by G13FEF.  
 If COPTS(1) = .TRUE., stationary conditions are enforced, otherwise they are not.  
 If COPTS(2) = .TRUE., the routine provides initial parameter estimates of the regression terms, otherwise these are provided by the user.
- 21: MAXIT – INTEGER *Input*  
*On entry:* the maximum number of iterations to be used by the optimization routine when estimating the GARCH( $p, q$ ) parameters. If MAXIT is set to 0 then the standard errors, score vector and variance-covariance are calculated for the input value of  $\theta$  in THETA; however the value of  $\theta$  is not updated.  
*Constraint:* MAXIT  $\geq$  0.
- 22: TOL – *real* *Input*  
*On entry:* the tolerance to be used by the optimization routine when estimating the GARCH( $p, q$ ) parameters.

- 23: WORK(LWORK) – *real* array *Workspace*  
 24: LWORK – INTEGER *Input*

*On entry:* the dimension of the array WORK as declared in the (sub)program from which G13FEF is called.

*Constraint:*  $LWORK \geq (NREG + 3) \times NUM + NPAR + 403$ .

- 25: IFAIL – INTEGER *Input/Output*

*On entry:* IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL  $\neq$  0 on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, NREG < 0,  
 or MN > 1,  
 or MN < 0,  
 or IQ < 1,  
 or IP < 0,  
 or NPAR  $\geq$  20,  
 or NPAR has an invalid value,  
 or LDC < NPAR,  
 or LDX < NUM,  
 or DIST  $\neq$  'N',  
 or DIST  $\neq$  'T',  
 or MAXIT < 0,  
 or NUM < max(IP, IQ).

IFAIL = 2

On entry,  $LWORK < (NREG + 3) \times NUM + NPAR + 403$ .

IFAIL = 3

The matrix  $X$  is not full rank.

IFAIL = 4

The information matrix is not positive definite.

IFAIL = 5

The maximum number of iterations has been reached.

IFAIL = 6

The log-likelihood cannot be optimised any further.

IFAIL = 7

No feasible model parameters could be found.

## 7 Accuracy

Not applicable.

## 8 Further Comments

None.

## 9 Example

This example program uses G05HMF to generate 1500 data points, with known process parameters  $\theta$  for the following two time-series:

- (i) A GARCH(1,1) sequence with normally distributed residuals.
- (ii) A GARCH(1,1) sequence with Student's  $t$ -distributed residuals.

Here G05HMF is initially called, with the output discarded, to eliminate 'start-up effects' in these sequences. The process parameter estimates,  $\hat{\theta}$ , are then obtained using G13FEF, and compared with their true values,  $\theta$ . Finally a four step ahead volatility estimate is computed using G13FFF.

### 9.1 Program Text

**Note:** the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      G13FEF Example Program Text
*      Mark 20 Release. NAG Copyright 2001.
*      .. Parameters ..
INTEGER          NOUT
PARAMETER       (NOUT=6)
INTEGER          NPARMX, NUM
real           ZERO
PARAMETER       (NPARMX=10,NUM=2000,ZERO=0.0e0)
INTEGER          NUM1, NREGMX
PARAMETER       (NUM1=3000,NREGMX=10)
*      .. Local Scalars ..
real          DF, FAC1, GAMMA, HP, LGF, MEAN, TOL, XTERM
INTEGER          I, IFLAG, IGEN, IP, IQ, K, LDX, LWK, MAXIT, MN,
+              NPAR, NPAR2, NREG, NT
LOGICAL          FCALL
CHARACTER       DIST
*      .. Local Arrays ..
real          BX(10), COVAR(NPARMX,NPARMX), CVAR(100),
+              ETM(NUM1), HT(NUM1+10), HTM(NUM1), PARAM(NPARMX),
+              RVEC(40), RWSAV(9), SC(NPARMX), SE(NPARMX),
+              THETA(NPARMX), WK(NUM1*3+NPARMX+NREGMX*NUM1+20*
+              20+1), X(NUM1,10), YT(NUM1+10)
INTEGER          ISEED(4)
LOGICAL          COPTS(2)
*      .. External Subroutines ..
EXTERNAL        G05HMF, G05KBF, G13FEF, G13FFF
*      .. Intrinsic Functions ..
INTRINSIC       real, SIN
*      .. Executable Statements ..

WRITE (NOUT,*) 'G13FEF Example Program Results'

ISEED(1) = 111
IGEN = 0

LWK = NUM1*3 + NPARMX + NREGMX*NUM1 + 1
```

```

NREG = 0
LDX = NUM1
DF = 5.1e0
GAMMA = 0.1e0
BX(1) = 1.5e0
BX(2) = 2.5e0
BX(3) = 3.0e0
MEAN = 4.0e0

DO 20 I = 1, NUM
  FAC1 = real(I)*0.01e0
  X(I,2) = 0.01e0 + 0.7e0*SIN(FAC1)
  X(I,1) = 0.5e0 + FAC1*0.1e0
  X(I,3) = 1.0e0
20 CONTINUE

MN = 1
NREG = 2
GAMMA = 0.1e0
IP = 1
IQ = 1
NPAR = IQ + IP + 1
PARAM(1) = 0.4e0
PARAM(2) = 0.1e0
PARAM(3) = 0.7e0
FCALL = .TRUE.

DIST = 'N'
CALL G05KBF(IGEN, ISEED)

CALL G05HMF(DIST, 200, IP, IQ, PARAM, GAMMA, DF, HT, YT, FCALL, RVEC, IGEN,
+          ISEED, RWSAV, IFLAG)

FCALL = .FALSE.
CALL G05HMF(DIST, NUM, IP, IQ, PARAM, GAMMA, DF, HT, YT, FCALL, RVEC, IGEN,
+          ISEED, RWSAV, IFLAG)

DO 60 I = 1, NUM
  XTERM = ZERO
  DO 40 K = 1, NREG
    XTERM = XTERM + X(I,K)*BX(K)
40  CONTINUE
  IF (MN.EQ.1) THEN
    YT(I) = MEAN + XTERM + YT(I)
  ELSE
    YT(I) = XTERM + YT(I)
  END IF
60 CONTINUE

IFLAG = -1

COPTS(1) = .TRUE.
COPTS(2) = .TRUE.

MAXIT = 100
TOL = 1.0e-5

DO 80 I = 1, NPAR
  THETA(I) = PARAM(I)*0.5e0
80 CONTINUE
THETA(NPAR+1) = GAMMA*0.5e0
IF (MN.EQ.1) THEN
  THETA(NPAR+MN+1) = MEAN*0.5e0
END IF
DO 100 I = 1, NREG
  THETA(NPAR+MN+1+I) = BX(I)*0.5e0
100 CONTINUE

NPAR2 = 2 + IP + IQ + MN + NREG
CALL G13FEF(DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, NPAR2, THETA, SE, SC,
+          COVAR, NPARMX, HP, ETM, HTM, LGF, COPTS, MAXIT, TOL, WK, LWK,

```

```

+           IFLAG)

WRITE (NOUT,*)
WRITE (NOUT,*) 'Normal distribution'
WRITE (NOUT,*)
WRITE (NOUT,*) '           Parameter           Standard           Correct'
WRITE (NOUT,*) '           estimates           errors           values'

DO 120 I = 1, NPAR
  WRITE (NOUT,99999) THETA(I), SE(I), PARAM(I)
120 CONTINUE

WRITE (NOUT,99999) THETA(NPAR+1), SE(NPAR+1), GAMMA

IF (MN.EQ.1) THEN
  WRITE (NOUT,99999) THETA(NPAR+2), SE(NPAR+2), MEAN
END IF

DO 140 I = 1, NREG
  WRITE (NOUT,99999) THETA(NPAR+MN+I+1), SE(NPAR+MN+I+1), BX(I)
140 CONTINUE

NT = 4
CALL G13FFF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)

WRITE (NOUT,*)
WRITE (NOUT,99998) 'Volatility forecast = ', CVAR(NT)
WRITE (NOUT,*)

DIST = 'T'
MEAN = 3.0e0

DO 160 I = 1, NUM
  FAC1 = real(I)*0.01e0
  X(I,2) = 0.01e0 + 0.7e0*SIN(FAC1)
  X(I,1) = 0.5e0 + FAC1*0.1e0
  X(I,3) = 1.0e0
160 CONTINUE

MN = 1
NREG = 2
GAMMA = 0.09e0
IP = 1
IQ = 1

NPAR = IQ + IP + 1
PARAM(1) = 0.05e0
PARAM(2) = 0.1e0
PARAM(3) = 0.8e0

FCALL = .TRUE.
ISEED(1) = 111
CALL G05KBF(IGEN,ISEED)

CALL G05HMF(DIST,200,IP,IQ,PARAM,GAMMA,DF,HT,YT,FCALL,RVEC,IGEN,
+         ISEED,RWSAV,IFLAG)
FCALL = .FALSE.
CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,FCALL,RVEC,IGEN,
+         ISEED,RWSAV,IFLAG)
CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,FCALL,RVEC,IGEN,
+         ISEED,RWSAV,IFLAG)

DO 200 I = 1, NUM
  XTERM = ZERO
  DO 180 K = 1, NREG
    XTERM = XTERM + X(I,K)*BX(K)
180 CONTINUE
  IF (MN.EQ.1) THEN
    YT(I) = MEAN + XTERM + YT(I)
  ELSE
    YT(I) = XTERM + YT(I)

```



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        END IF
200 CONTINUE

        IFLAG = -1

        MAXIT = 100
        TOL = 1.0e-5

        DO 220 I = 1, NPAR
            THETA(I) = PARAM(I)*0.5e0
220 CONTINUE

        THETA(NPAR+1) = GAMMA*0.5e0

        THETA(NPAR+2) = DF*0.65e0

        IF (MN.EQ.1) THEN
            THETA(NPAR+2+MN) = MEAN*0.5e0
        END IF
        DO 240 I = 1, NREG
            THETA(NPAR+2+MN+I) = BX(I)*0.5e0
240 CONTINUE

        COPTS(1) = .TRUE.
        COPTS(2) = .TRUE.
        HP = 0.5e0

        NPAR2 = 3 + IP + IQ + MN + NREG

        CALL G13FEF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,NPAR2,THETA,SE,SC,
+                COVAR,NPARAMX,HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,
+                IFLAG)

        WRITE (NOUT,*)
        WRITE (NOUT,*) 'Student t-distribution'
        WRITE (NOUT,*)
        WRITE (NOUT,*) '          Parameter          Standard          Correct'
        WRITE (NOUT,*) '          estimates          errors          values'

        DO 260 I = 1, NPAR
            WRITE (NOUT,99999) THETA(I), SE(I), PARAM(I)
260 CONTINUE

        WRITE (NOUT,99999) THETA(NPAR+1), SE(NPAR+1), GAMMA

        WRITE (NOUT,99999) THETA(NPAR+2), SE(NPAR+2), DF

        IF (MN.EQ.1) THEN
            WRITE (NOUT,99999) THETA(NPAR+2+MN), SE(NPAR+2+MN), MEAN
        END IF

        DO 280 I = 1, NREG
            WRITE (NOUT,99999) THETA(NPAR+2+MN+I), SE(NPAR+2+MN+I), BX(I)
280 CONTINUE

        NT = 4
        CALL G13FFF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)

        WRITE (NOUT,*)
        WRITE (NOUT,99998) 'Volatility forecast = ', CVAR(NT)
        WRITE (NOUT,*)
        STOP
*

99999 FORMAT (1X,3F16.4)
99998 FORMAT (1X,A,F12.4)
END

```

## 9.2 Program Data

None.

## 9.3 Program Results

G13FEF Example Program Results

Normal distribution

Parameter estimates	Standard errors	Correct values
0.3707	0.0780	0.4000
0.1036	0.0257	0.1000
0.7079	0.0414	0.7000
0.1188	0.0370	0.1000
4.0998	0.0950	4.0000
1.4251	0.0592	1.5000
2.2604	0.0683	2.5000

Volatility forecast = 1.7067

Student t-distribution

Parameter estimates	Standard errors	Correct values
0.0378	0.0091	0.0500
0.0834	0.0253	0.1000
0.8108	0.0270	0.8000
0.1163	0.0377	0.0900
5.7528	0.6255	5.1000
2.9676	0.0357	3.0000
1.4889	0.0226	1.5000
2.5161	0.0303	2.5000

Volatility forecast = 0.5981

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