

S13AAF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S13AAF returns the value of the exponential integral $E_1(x)$, via the routine name.

2 Specification

```

real FUNCTION S13AAF(X, IFAIL)
  INTEGER          IFAIL
  real            X

```

3 Description

The routine calculates an approximate value for

$$E_1(x) = \int_x^\infty \frac{e^{-u}}{u} du, \quad x > 0.$$

For $0 < x \leq 4$, the approximation is based on the Chebyshev expansion

$$E_1(x) = y(t) - \ln x = \sum_r' a_r T_r(t) - \ln x,$$

where $t = \frac{1}{2}x - 1$.

For $x > 4$,

$$E_1(x) = \frac{e^{-x}}{x} y(t) = \frac{e^{-x}}{x} \sum_r' a_r T_r(t),$$

where $t = -1.0 + 14.5/(x + 3.25) = \frac{11.25-x}{3.25+x}$.

In both cases, $-1 \leq t \leq +1$.

To guard against producing underflows, if $x > x_{hi}$ the result is set directly to zero. For the value of x_{hi} see the Users' Note for your implementation.

4 References

- [1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

5 Parameters

- 1: X — **real** *Input*
On entry: the argument x of the function.
Constraint: $X > 0.0$.
- 2: IFAIL — INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

The routine has been called with an argument less than or equal to zero for which the function is not defined. The result returned is zero.

7 Accuracy

If δ and ϵ are the relative errors in argument and result respectively, then in principle,

$$|\epsilon| \simeq \left| \frac{e^{-x}}{E_1(x)} \times \delta \right|$$

so the relative error in the argument is amplified in the result by at least a factor $e^{-x}/E_1(x)$. The equality should hold if δ is greater than the *machine precision* (δ due to data errors etc.) but if δ is simply a result of round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation and round-off.

The behaviour of this amplification factor is shown in the following graph:

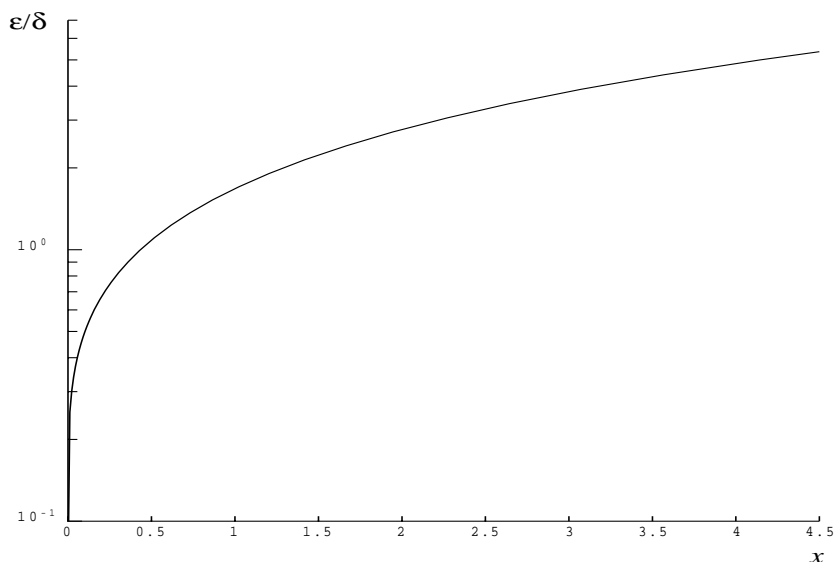


Figure 1

It should be noted that, for small x , the amplification factor tends to zero and eventually the error in the result will be limited by *machine precision*.

For large x ,

$$\epsilon \sim x\delta = \Delta,$$

the absolute error in the argument.

8 Further Comments

None.

9 Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      S13AAF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real            X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      real            S13AAF
      EXTERNAL         S13AAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S13AAF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      X          Y          IFAIL'
      WRITE (NOUT,*)
20     READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S13AAF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
40     STOP
*
99999  FORMAT (1X,1P,2e12.3,I7)
      END

```

9.2 Program Data

```

S13AAF Example Program Data
      2.0
     -1.0

```

9.3 Program Results

S13AAF Example Program Results

X	Y	IFAIL
2.000E+00	4.890E-02	0
-1.000E+00	0.000E+00	1
