Test of a real time cyclostationary RFI detector for radio astronomy

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Abstract

Radio frequency Interference (RFI) could seriously degrade the observations in the radio astronomy bands, hence suitable methods to detect and mitigate them have to be implemented in the receiver.

In this report the implementation on a FPGA of an RFI detector for radio astronomy based on the cyclostationarity is described. After a brief overview on the basic principles of RFI detection and cyclostationarity, performance in terms of detection and false alarm probability has been derived using a base-band BPSK as a test signal. Moreover the false alarm probability derived by theoretical analysis has been compared to the experimental results.

Finally an uplink GSM signal has been acquired to further test our detection approach. However, the previous method requires the knowledge of the cyclic frequency to be tested and in the GSM case the cyclic frequency depends on the carrier frequency that is unknown. To overcome this issue a more general method, based on the cyclic autocorrelation function (CAF) estimation, is proposed. In particular, the CAF estimation of the GSM acquired signal is partially carried out in the FPGA, because the FFT is calculated off-line. The comparison between the GSM CAF estimate and the CAF related to a simulated radio source shows that this approach can be employed as detector also with this kind of interference. In the future we will study the real time implementation on a FPGA and performance evaluation of this approach.

1 Introduction

The next generation radio telescope such as SKA, LOFAR is intended to be at least two magnitude more sensitive than the current generation of radio telescopes. The increased sensitivity makes such a telescope more vulnerable to radio unwanted emissions (Radio frequency Interference, RFI), mainly caused by the increased spectrum utilization due to exponential growing of wireless and satellite communication systems.

Regulation can protect particular windows of the radio spectrum but many observations need to access to a wider spectral range which can be contaminated by other spectrum users, because the increased sensitivity of future radio telescope will permit the observation of extremely faint and distant radio sources which exhibit large amounts of red shift, hence overlapping with unprotected bands in the radio spectrum.

Hence, real time detection and mitigation of RFI is becoming a concern of increasing importance for radio astronomy. In the literature several signal processing methods providing reliable real time RFI detection and mitigation, operating at various stages of the receiver chain, have been presented.

The focus of this report is the description of a signal processing method which exploits the cyclostationarity of the RFI produced by a digital communication signal to distinguish it from noise and from the radio sources of interest [1]. The detected RFI can be suppressed with a time excision if it is bursty.

The report is structured as follows: the first two sections will present the basic concepts of RFI detection and of the Cyclostationarity. Then an implementation on a FPGA of a real time cyclostationary detector algorithm will be presented including the performance in terms of detection probability of for a simple base-band test signal.

A comparison between the false alarm probability derived by a theoretical analysis and that obtained by experimental results is also presented. Finally, after describing the acquisition GSM uplink signal, the cyclostationary detection of this kind of interference has been investigate. However because the carrier frequency of the GSM signal is *a priori* unknown the *coherent detection* can not be performed. As a consequence the previous method which requires the knowledge of the cyclic frequency that in the GSM case depends on the carrier frequency can not be employed.

An alternative method based on the cyclic autocorrelation function (CAF) estimation is then outlined. In particular, the CAF estimation of the GSM acquired channel is partially carried out in the FPGA, because the FFT is calculated off-line. The comparison between the GSM CAF estimate and the CAF related to a simulated radio source shows that this approach can be employed as detector also with this kind of interference.

2 The RFI detection: Hypothesis testing

In this section a brief overview of the RFI detection basic concepts will be presented. In general, the signal received from a single dish antenna, $r(t)$ can be written as:

$$r(t) = n(t) + x(t) \tag{1}$$

where $n(t)$ is the Additive White Gaussian Noise (AWGN) term which includes the receiver noise, sky noise and the radio astronomical source, while $x(t)$ refers to the RFI. The RFI detection is a classical statistical signal processing problem and it can be formulated as the following, well known, hypothesis testing:

$$\begin{cases} \mathrm{H}_0 : r(t) = n(t) \\ \mathrm{H}_1 : r(t) = x(t) + n(t) \end{cases} \tag{2}$$

where H_0 is the *null hypothesis*, and H_1 is the *alternative hypothesis*. Therefore an hypothesis testing consists in the analysis of a set of samples of the received signals (that is a random variable representing the statistical test) in order to assess the presence of RFI in the received signal, i.e, to discriminate between the hypothesis H_0 and H_1.

Moreover to evaluate the performance of an RFI detector we define the detection probability of RFI P_d, as the probability that statistical test gives a correct decision when H_1 is true. On the other hand we denotes with P_{fa} the false alarm probability, i.e., the probability that statistical test gives a wrong decision when H_0 is true.

The usual method to perform RFI detection is based on power estimation. However, although its reduced implementation complexity, it needs a definition of a threshold in order to perform reliable RFI detection. Hence in presence of non stationary context (i.e., time fluctuations of the instrumental conditions), this method is not effective.

In order to overcome this drawback a power independent method has been studied. This detection method exploits the cyclostationary properties, which exists in most man-made communication signals, hence enabling the detection of RFI derived by man-made communication signals.

3 The cyclostationarity

In this section a brief recap of the basic principles and definitions associated with cyclostationary processes is presented. In general, a continuous time random process $x(t)$ is said to be wide sense *non conjugate* cyclostationary if its time varying autocorrelation

$$R(t,\tau) = E\left\{x(t)x^*(t+\tau)\right\} = E\left\{x(t+T_c)x^*(t+\tau)\right\} \tag{3}$$

is periodical in t, where τ denotes the time lag and T_c the periodicity (often called *hidden periodicity*). A signal $x(t)$ exhibits *conjugate cyclostationarity* if we have:

$$R(t,\tau)^* = E\left\{x(t)x(t+\tau)\right\} = E\left\{x(t+T_c)x(t+\tau)\right\} \tag{4}$$

For modulated signals, the periods of cyclostationarity correspond to carrier frequencies, pulse rates, spreading code repetition rates, time-division multiplexing rates, and so on. Hence, the autocorrelation function of cyclostationary signal can be developed into a Fourier series:

$$R(t,\tau) = \sum_{\alpha} R_x^{\alpha}(\tau)e^{j2\pi\alpha t} \tag{5}$$

where the Fourier coefficients are:

$$R_x^{\alpha}(\tau) = \frac{1}{T_c}\int_{-\infty}^{\infty} R(n,\tau)e^{j2\pi\alpha n}\,dt \tag{6}$$

The function $R_x^{\alpha}(\tau)$ is called the cyclic autocorrelation function (CAF). If the autocorrelation is a function exactly periodical with a period T_c , The CAF presents nonzero values at $\{\alpha = k/T_c\,, k \geq 1\,, k \in \mathbb{N}\}$. The discrete form of (6) can be expressed as:

$$R_x^{\alpha}(\tau) = \sum_{n=-\infty}^{n=\infty} R(n,\tau)e^{j2\pi\alpha n} \tag{7}$$

An estimation frequently used of (7) is the following:

$$\hat{R}^{\alpha}(\tau) \simeq \sum_{n=0}^{N} x(n)x^*(n+\tau)e^{j2\pi\alpha n} \tag{8}$$

For a non-cyclostationary signal, such as the receiver noise $n(t)$ modelled as AWGN (Additive White and Gaussian), the energy of the CAF is concentrated at $\alpha = 0$.

However it can be easily shown (see also 4.3) that also for a stationary radio astronomical source the $R_x^{\alpha}(\tau)$ has only a peak around the cyclic frequency $\alpha = 0$.

Let $s(t)$ be a typical radio astronomical source, we can assess that $s(t)$ is a Gaussian, stationary signal. In particular, $s(t)$ can be considered filtered white noise. Eq. (6) then becomes:

$$R_x^{\alpha}(\tau) = \frac{1}{T_c}\int_{-\infty}^{\infty} R(t,\tau)e^{j2\pi\alpha t}\,dt = \frac{R(\tau)}{T_c}\int_{-\infty}^{\infty} e^{j2\pi\alpha t}\,dt = \frac{R(\tau)}{T_c}\delta(\alpha) \tag{9}$$

where $\delta(\tau)$ is the Dirac delta function and $R_s(t,\tau) = R_s(\tau)$ for the hypothesis of stationarity.

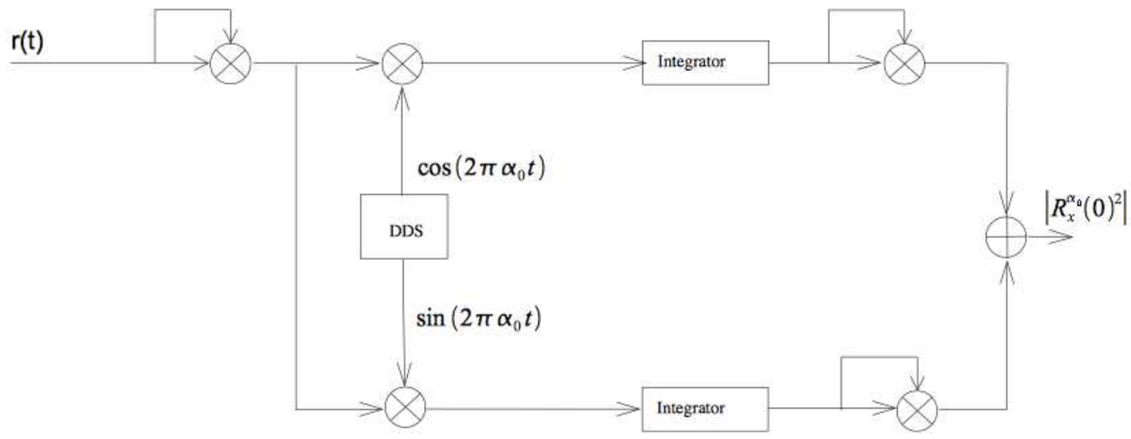


Figure 1: block scheme of $\hat{R}_x^{\alpha_0}(0)$ computation

3.1 The proposed algorithm and FPGA implementation

The implementation outlined here relies on the estimation of the $R_x^{\alpha}(\tau)$ proposed in (8). In particular, assuming known the cyclic frequency $\alpha_1 = 1/Tc$ that we want to test, a statistical test T_α, similar to that one proposed in [1] is presented here.

$$T_\alpha = \left|\frac{\hat{R}_x^{\alpha_1}(0)}{\hat{R}_x^{\alpha_0}(0)}\right|^2 \simeq \left|\frac{\sum_{n=0}^{N} x(n)x^*(n+\tau)e^{j2\pi\alpha_1 n}}{\sum_{n=0}^{N} x(n)x^*(n+\tau)e^{j2\pi\alpha_0 n}}\right|^2_{\tau=0} = \left|\frac{\sum_{n=0}^{N} x(n)^2 e^{j2\pi\alpha_1 n}}{\sum_{n=0}^{N} x(n)^2 e^{j2\pi\alpha_0 n}}\right|^2 \tag{10}$$

is a simple ratio between $\hat{R}_x^{\alpha_1}(\tau)$ evaluated at the frequency bin related to the first harmonic $\alpha = 1/T_c$ and $\hat{R}_x^{\alpha_0}(\tau)$ which is the CAF evaluated at the frequency bin α_0, not multiple of α;

The test defined by (10) has been written in VHDL and then implemented in the FPGA Altera stratix via the sotware Quartus8. The Fig. 1 shows with a block scheme how is carried out the computation of $\hat{R}_x^{\alpha_0}(0)$. The incoming received signal $s(t)$ is squared then divided in two branches: real and imaginary. In the real one $s(t)$ is multiplied with the $cos(2\pi\alpha_0 t)$, (the real output of the DDS programmed with α_0), then it is integrated and squared to produce the squared value of the real part $\hat{R}_x^{\alpha_0}(0)$.

The other branch produces the squared value of the imaginary part. Finally, these two modules squared are summed to produce the squared modulus of $\hat{R}_x^{\alpha_0}(0)$.

An analogue block is used to calculate $\hat{R}_x^{\alpha_1}(0)$, which after multiplication with a proper threshold is compared to $\hat{R}_x^{\alpha_0}(0)$, to provide T_α.

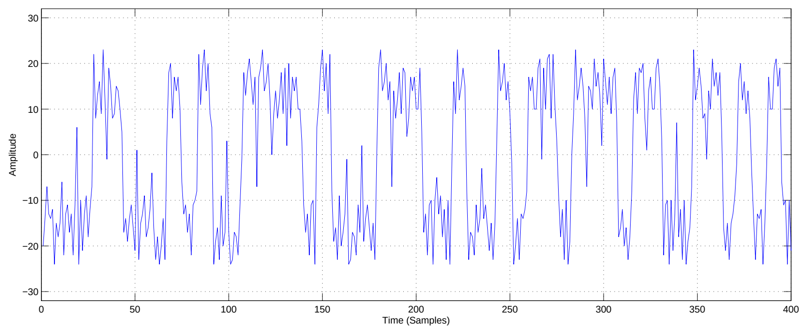
3.1.1 Test Signal Generation: BPSK and AWGN

The performance evaluation of the proposed algorithm has been carried out with the same interference employed in [1], i.e., a Binary Phase Shift Keying (BPSK) signal, with the pulse shape given by a Tukey window with the ratio of taper to constant sections equal to 0.1 and the bit time of the BPSK equal to 1μs, hence with a cyclic frequency $\alpha_1 = 1/T_b$ equal to 10 MHz.

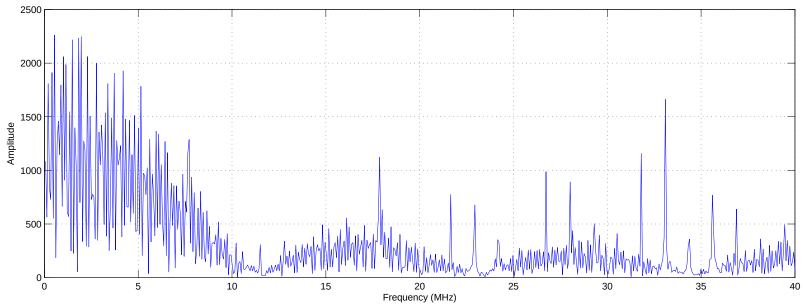
Signals which belongs to the class of BPSK, but generally employing up to 8 bit/symbol are used in digital radio link in the frequency range 4 - 40 GHz, hence they could be interfere to radio astronomical observations.

The test signals (i.e., the AWGN, and the interference) have been generated using the VHDL-coder, a tool developed in the Matlab-Simulink environment which generates VHDL code simply from a block scheme built in the Simulink environment. First we generate a Simulink model of BPSK signal shaped with a Tukey window using the Pseudonoise sequence generator provided by the Simulink library. Then the AWGN noise is generated summing five PN modules, in order to approximate a Gaussian signal.

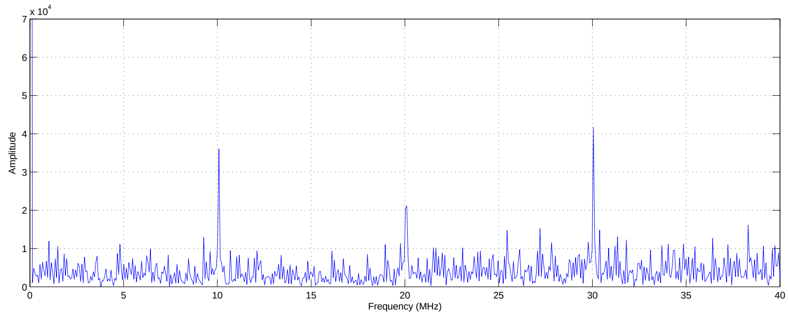
The modules related to the test signals have been embedded in a VHDL top level design which includes as DUT (Device Under Test) the detection algorithm. The top level includes also two counters: *H1* which



(a) BPSK: Time domain with AWGN



(b) BPSK with AWGN: Spectrum



(c) BPSK with AWGN: CAF

Figure 2: Test signal BPSK noisy, INR=10 dB

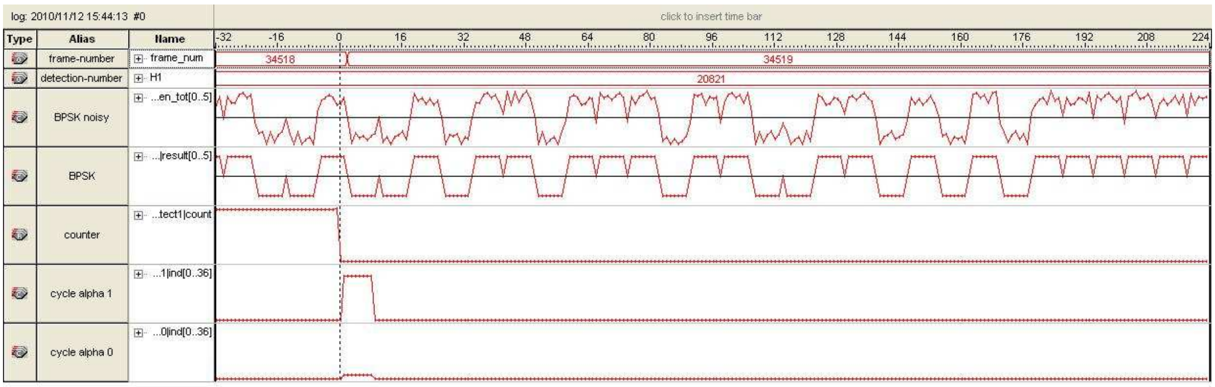


Figure 3: Signals highlighted in simulation, when the INR = 10 dB

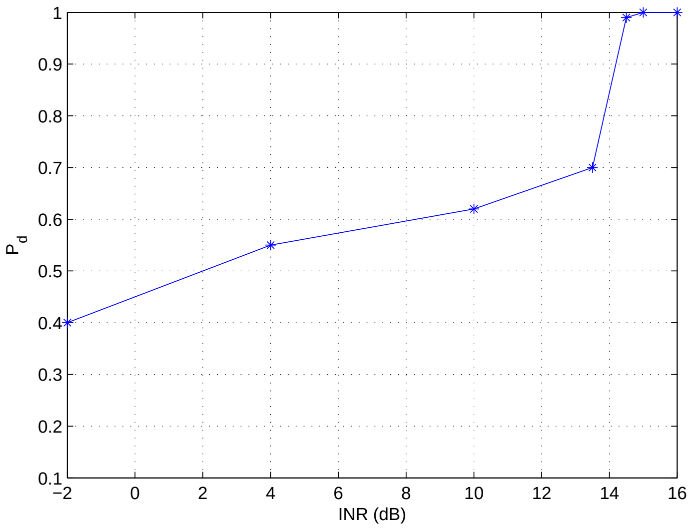


Figure 4: P_d vs INR

takes into account of the number of correct detections and *Frame Number* which is the counter related to the number of test performed: the ratio $H1/FrameNumber$ gives an approximation of P_d

The most important signals of the the FPGA implementation have been illustrated in Fig. 3 (a screenshot of the Quartus signal tap logic analyzer) showing the waveforms used in performance test.

In particular Fig. 3 shows the frame number, H_1, the counter signal, which triggers the integration (the integration period was set to 32768 samples), and the signal ind0 and ind1 representing $\hat{R}_x^{\alpha_0}(0)$ and $\hat{R}_x^{\alpha_1}(0)$ respectively.

The ratio of ind1 and ind0, compared to the threshold is therefore the test statistic. Fig. 3 also shows the test signal labelled with BPSK and the signal named BPSK noisy which represents the sum of BPSK and the AWGN (the total test signal).

Fig. 2a,b and c respectively shows the time domain test signal (acquired by the Quartus signal tap logical analyzer), its spectrum and its CAF, plotted with Matlab.

In particular the CAF depicted in Fig. 2c shows more clearly the presence of the cyclostationary interference with respect to the Fig. 2b due to the peaks at frequencies multiple of 10 MHz, i.e., $1/T_b$.

Fig. 4c shows the performance of our method in terms of detection probability (P_d) as a function of the Interference to Noise ratio (INR).

The resources employed for the algorithm implementation are illustrated in the table below:

Modulo	logic cells	LC Registers	M4Ks	DSP
RFI detect	761	684	2	21
CIC	69	68	0	0

Table 1: Resources employed for the algorithm

3.1.2 False Alarm Probability Computation

An important feature of RFI detector is the False Alarm Probability P_{fa}. P_{fa} has to be the lowest possible to guarantee the reliability of the method. In this scenario the input signal is only the noise e.g., producted by a noise generator.

A theoretical analysis of the P_{fa} of the method can be found in [3]. The Fig. 5 shows the P_{fa} as a function of the threshold compared with the theoretical analysis, highlighting a good match between experimental results and theory.

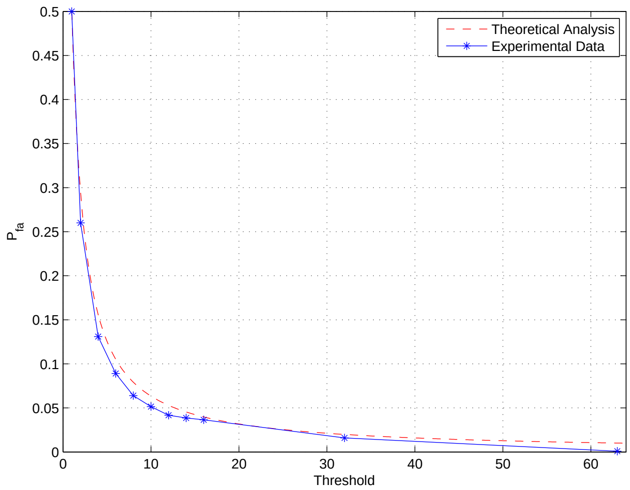


Figure 5: P_{fa} vs T_h

4 RFI detection with real data: the GSM detection

In this section a feasible implementation of a cyclostationary detection of a real RFI will be discussed. In particular, we focus on the uplink signal of the Global System for Mobile Communications (GSM) whose cyclostationary detection (in base-band case) has been theoretically shown in [2]. The acquisition system which consist of a RF front-end and a back-end, is schematically depicted in the Fig. 6

4.1 The acquisition system: front-end

The front-end is constituted by an isotropic antenna, followed by a band-pass filter as depicted in Fig. 6

The band-pass filter ensuring protection from interferences and developed in strip-line technology has the periodic frequency response described in Fig. 7, measured with a vector analyzer. The band-pass filter is followed by a ZLW-11 mixer, downconverting the incoming signal with a local oscillator LO set at 900 MHz, since the uplink 200 KHz bandwidth GSM signal spans in the 890-915 MHz band. Then we have a low-pass filter SLP 50+ with a DC- 48 MHz bandwidth. Due to the proximity of the cellular phone to the antenna, it is not necessary to use an amplifier.

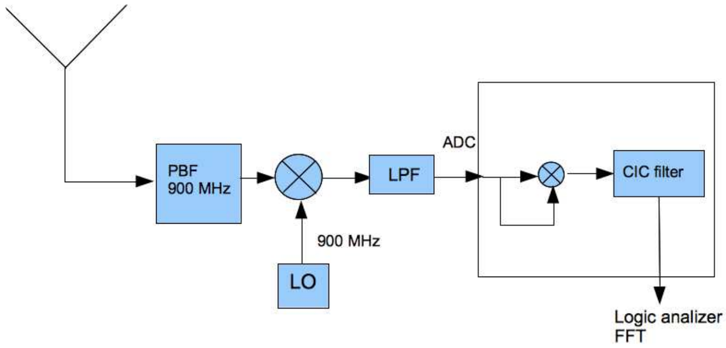


Figure 6: GSM uplink acquisition scheme

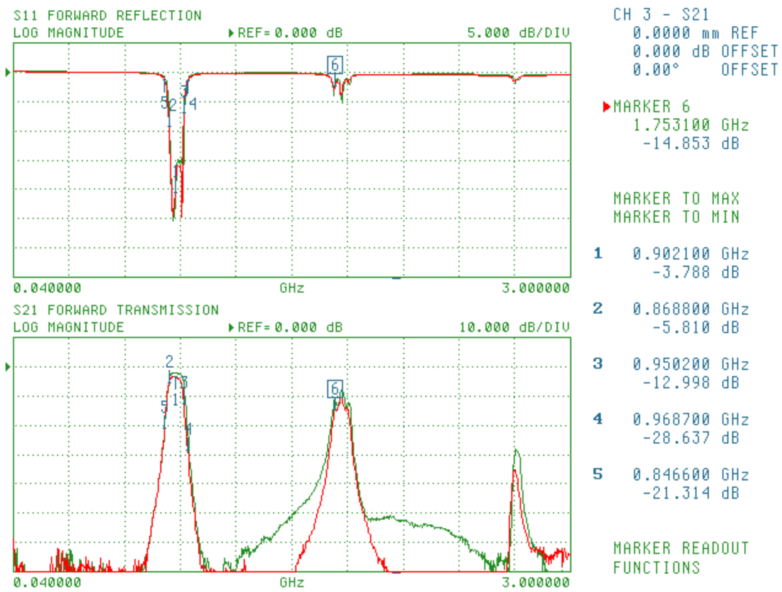


Figure 7: Filtro 900MHz

4.2 The acquisition system: back-end and the CAF estimation

The output of the front-end entering in the Altera Stratix FPGA is sampled at 80 MHz and quantized with 12 bits. Then this input signal has been captured by the Signal Tap Logical Analyzer, decimated to 4 MHz and processed by an FFT off-line using Matlab, to derive a spectrum of the GSM signal (Fig. 8)

Now, to illustrate the back-end, we recall that the discrete time varying correlation defined in (4) (over M samples) can be written as:

$$R(n, \tau) = \sum_{m=n}^{M+n} x(m)^* x(m + \tau) \tag{11}$$

where n is the time index.

In particular, we are interested to evaluate (11) for $\tau = 0$, assuming that $x(n)$ be a real signal. Therefore the back-end depicted in Fig. 6 process the input signal $x(n)$ to implements $R(n, 0)$ with a multiplier and CIC filter (a boxcar, which implements low-pass filters and decimation by a factor of 20). The CAF is finally estimated as:

$$\hat{R}_x^\alpha(0) = \mathcal{F}\left[R(n, 0)\right] = \sum_{n=0}^{N-1} \left[\sum_{m=n}^{M+n-1} x(m)^2\right] e^{j2\pi\alpha n} \tag{12}$$

with an N points off-line FFT of the signal $R(n, 0)$ previously acquired by the Signal Tap Logical Analyzer. It is important to notice that in this case the CAF is computed without using the approximated form.

4.3 Comparison between the CAF of GSM signal acquired and the CAF of a simulated radio-source

The method illustrated in section 3.1 has the drawback that it requires the exact knowledge of the cyclic frequency to be tested. This frequency can not be always available, for instance, because the cyclic frequency of a modulated signal depends on the carrier frequency that in general might be not known *a priori*.

In the GSM case, for instance, the exact carrier frequency of the channel used in uplink is unknown, as a consequence the cyclic frequency is shifted in frequency by an unknown offset and the ratio test proposed in sec.3.1 can not be performed.

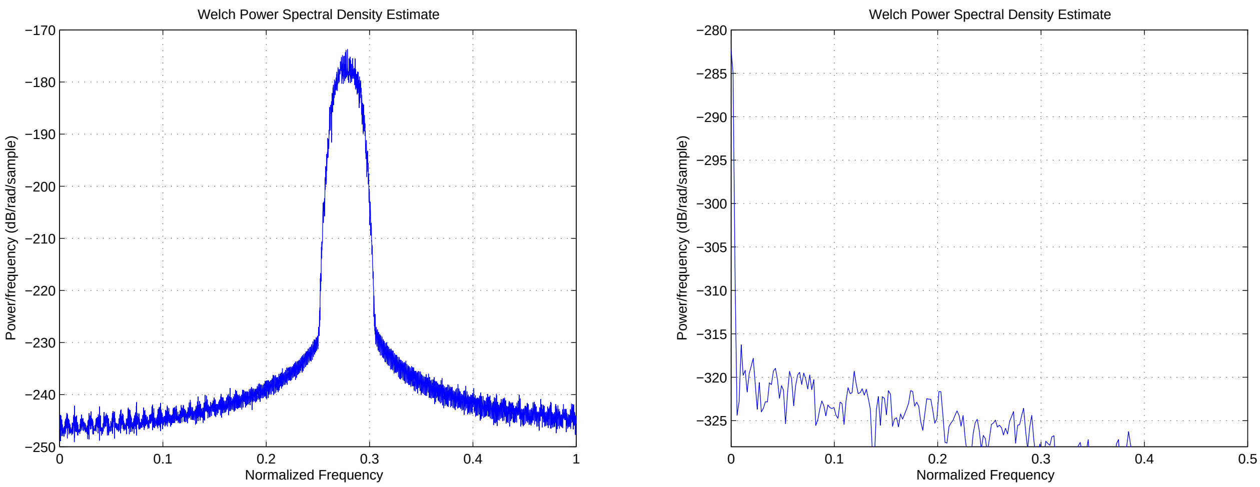
Furthermore, without the knowledge of the GSM uplink carrier frequency, it is not possible to perform a coherent detection, hence obtaining the real and imaginary part of received signal (the information content, or complex envelope). As a consequence, the cyclostationary detection has to be limited to the *non-coherent* cyclostationary detection implementation.

As shown in [2], the GSM signal, which is a GMSK (Gaussian Minimum Shift Keying) modulated signal, exhibits only *conjugate cyclostationarity*, hence lowering the detection performance of a *non-coherent* implementation, as in our case.

However, Fig. 9 shows that also detection *non-coherent* cyclostationary of the GSM received signal can be more effective than simple spectral analysis. In particular, in the Fig. 9 (d) that shows the CAF estimated as illustrated in sec.4.2, two peaks are clearly visible. The distance between the two peaks is about 270 KHz, (approximatively the bandwidth of the GSM signal, or the bit-rate) and the center-frequency is the result of the downconversion and the decimation of the signal.

This CAF of a simulated stationary radio source spectrum is depicted in the Fig. 8 (b), (while Fig. 8 (a) represents its spectrum). Due to the stationarity, the energy of the CAF is concentrated around zero frequency.

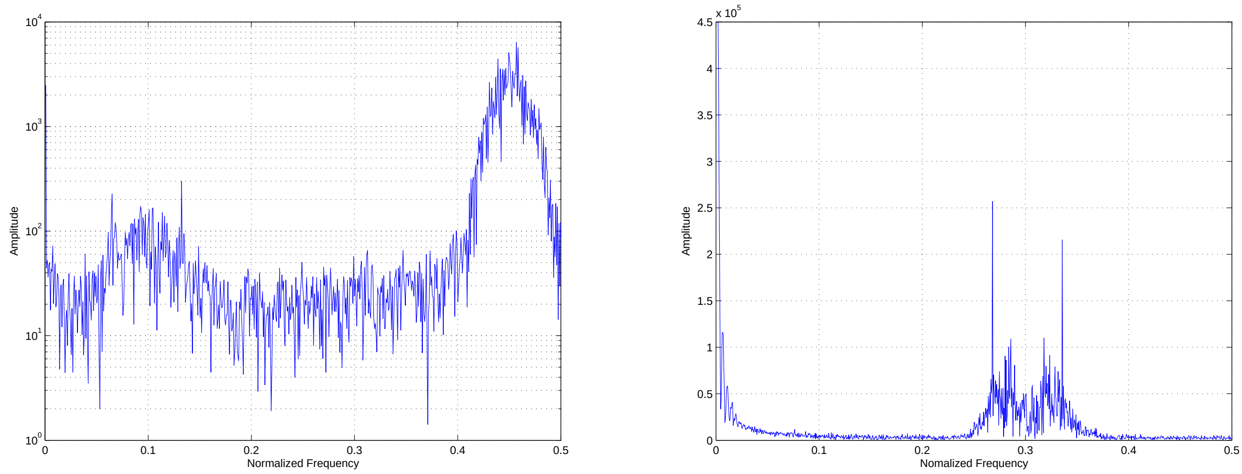
Hence comparing Fig. 8(b) and Fig. 9(b) , i.e., the CAF of radio source and the CAF of the GSM signal, the presence of interference can be inferred observing the presence of energy in the CAF at nonzero frequency (Fig. 9(b))



(a) Radio Source PSD

(b) Radio Source estimated CAF

Figure 8: PSD and CAF of a simulated Radio Source (frequency normalized to sampling frequency)



(a) GSM PSD (b) GSM estimated CAF

Figure 9: PSD and CAF of GSM uplink signal acquired with the back-end described (Frequency normalized to the decimated sampling frequency of 4 MHz)

5 Conclusions

In this report an implementation on a FPGA of a signal processing method based on the cyclostationarity to detect an RFI due to a radio digital communication signal has been tested. Performance results in terms of probability of detection for a base-band BPSK signal (hence, with the knowledge of the cyclic frequency to be tested) in AWGN environment has been presented.

A comparison between the false alarm probability derived by a theoretical analysis and the same probability obtained by experimental results it is also presented In order to test this approach to a real RFI, an uplink GSM signal has been acquired.

However GSM is a modulated signal with a carrier frequency not known *a priori* and because the cyclic frequencies depend on the carrier frequency, the same statistical test employed in the base-band BPSK case can not be used.

Furthermore, for the same reason, *coherent detection* can not be performed and the CAF estimation has to be limited to *non-coherent* implementation, although the *conjugate* CAF estimation exhibited by the phase modulated signals likewise GSM would require *coherent detection*.

However, the CAF of a GSM uplink signal acquired, compared to the CAF of a simulated radio source, shows the possibility of distinguish the two signals.

In the future we will study the implementation and the performance evaluation of a detector based on real time CAF estimator, where also the FFT is included in the FPGA.

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Contents