SKA Project Series
Quantization noise, linearity and spectral whitening in the LFAA quantizer

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Abstract

The LFAA ADC must provide high dynamic range, high linearity, good performance in presence of RFI and low added noise. These specifications may be seriously degraded by the large variation in spectral power density across the input band. This aspect is discussed here, together with the advantages and disadvantages of a whitening equalization filter.

List of acronyms

ADC: Analog to Digital converter

ADU: Analog to Digital converter quantization Unit (step)

ENOB: Equivalent Number of Bits

LFAA: Low Frequency Aperture Array

LNA: Low Noise Amplifier

RFI: Radio Frequency Interference

RMS: Root Mean Squared

1 Introduction

The LFAA digitizer is specified for 8 bit quantization, as 8 bits are usually sufficient to achieve the required dynamic range with a noiselike signal.

The digitization process must guarantee[1]:

- high dynamic range, for accurate and deep mapping. SKA level 1 specification requires the system to be able to accurately map sources with a flux 50 dB below those of the large sources in the field of view.
- high linearity (low intermodulation products), to a level at least below the instantaneous noise level in the fine channelization bandwidth (≈ 1 kHz) of the correlator.
- low added noise. It is not clear from level 1 specifications if the ADC noise is part of the digital signal processing noise (2% of the total, SKA1-SYS_REQ-2678) or part of the receiver noise (15-20% of the total). Correspondingly, the quantization noise could be either 0.5% or up to several % of the total. In this report we will consider the most stringent specification, of an added quantization noise limit of 0.5% of the total.

RFI immunity is included in the linearity specification. As long as the system is linear, RFI affected portions of the spectrum can be safely deleted during signal processing, without contaminating *good* portions of the spectrum.

The high dynamic range also calls for a very good linearity of the quantization process, even in the absence of strong interfering signals. This limits the maximum signal level at the ADC input, as discussed in section 2.1.

The ADC adds noise that is flat in frequency, with a total amplitude of a fraction of the quantization step. As the input signal to the LFAA is strongly frequency dependent, with about 20 dB of difference between the extremes of the observed bandwidth, this may cause certain portions of the spectrum to have a lower SNR. The overall performance can be improved by using a whitening filter in the amplifier chain. This adds some complexity to the system, so a clear evaluation of the advantages is required [4]. The receiver chain bandpass (especially the LNA) contributes to the spectral shaping of the signal and must be considered in the analysis.

An analysis of the receiver chain performance, especially the LNA bandpass gain, has been derived from [5]. The RFI characterization of the site, as seen by a prototype antenna, is reported in [6].

2 Constraints and assumptions

The digitizer noise depends on several parameters in the RX chain system and environment. In this section we will briefly review these parameters, making reasonable assumptions on ithe range of these parameters.

2.1 Linearity for a digitized signal

Several factors limit the ADC linearity performances. Any quantization process is intrinsically nonlinear. The stochastic nature of the signal to be analyzed introduces a strong dithering, greatly increasing the dynamic range and the linearity of the quantization process. In particular, the usual limit of roughly $6n_b$ dB of dynamic range for a n_b bits ADC is not valid, and spurious free dynamic ranges well in excess of 100 dB are routinely achieved even with 1 bit quantization.

The quantity observed by a radiotelescope is the averaged product $< x_1 x_2 >$ of two noise-like signals, x_1 and x_2. Here the angle brackets $<>$ indicate the temporal average operation. We assume that both signals have a Gaussian statistics, the same power spectrum, and a cross correlation coefficient that depends on the frequency, $\rho(\nu)$. The correlation coefficient of two signals is defined as:

$$\rho = \frac{\langle x_1 x_2 \rangle}{\sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle}} \tag{1}$$

ρ is in the range of -1 and 1, and indicates the fraction of the noise that is common to the two signals. The unquantized correlation product is then

$$r = \langle x_1 x_2 \rangle = \rho \sqrt{\langle x_1^2 \rangle \langle x_2^2 \rangle} \tag{2}$$

The digitization process must preserve the stochastic properties of the signal, and must not introduce nonlinearities even when a deterministic (e.g a sinusoid) signal is added to the noise. This can be analyzed by studying the properties of the product of the two digitized signals X_1, X_2, where the uppercase quantity X_i is the digitized version of x_i, i.e. the function

$$R(r) = \langle X_1 X_2 \rangle \tag{3}$$

$R(r)$ is a function of the correlation product r, of the RMS amplitude of the two original signals x_1, x_2, and of the number of quantization levels in the ADC. For an ideal ADC, with infinite bits, the unquantized and quantized correlation products should be identical, i.e. $R(r) = r$. For a real ADC this function deviates from the identity relation both for a gain error and for nonlinear terms. To study these effects, it is possible to expand the relationship in a Taylor expansion, and study the magnitude of the linear term (gain) and of the first terms in the expansion. The problem is discussed in [2] and, with a more pragmatic approach, in [3]. The main results of this analysis is that the quantization introduces three main effects:

- The nonlinear term in $R(r)$ is usually negligible if the amplitude of the original signal is less than 1/6 of the total quantization range, and more than about 3 ADC units (ADU).
- The "ADC gain", i.e. the slope of $R(r)$ with respect to the ideal, continuous case, is less than 1, mainly due to clipping.
- The ADC increases the total noise in the system.

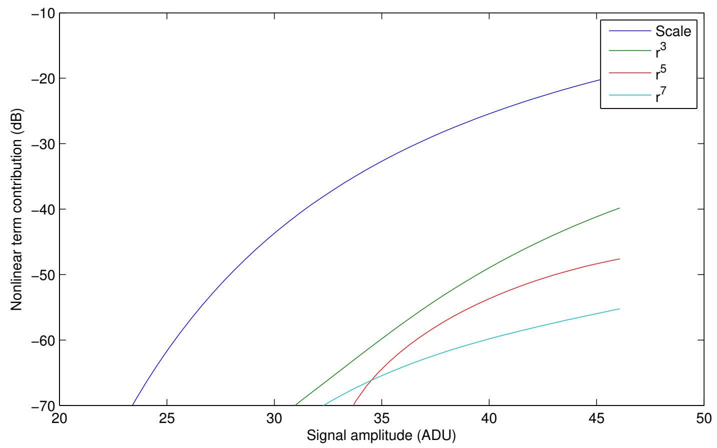

Figure 1: Nonlinearity terms for a 8 bit quantization, $\rho = 0.5$

2.1.1 Nonlinearities in $R(r)$

The function $R(r)$ is odd, $R(-r) = -R(r)$, and therefor its Taylor expansion contains only odd terms. As usually r is small, the dominant term is that in r^3. Assuming an upper limit for $\rho = 0.5$, one can estimate the relative importance of this term with respect to R, as a function of the number of bits n_b in the quantization and of the signal level, in ADC units (ADU). A correlation coefficient of 0.5 is rather high, but can be reached for strong emission and antennas close together, in the same station. It can be very high also in the presence of strong interfering signals, thus affecting the generation of intermodulation terms.

The analysis in [2] shows that this is negligible (below -50 dB) if the RMS amplitude of the signal is comprised between about 3 ADU, and 1/6 of the total ADC range 2^{n_b}. For less than 5 bit these conditions are never satisfied, and a linearization correction is required (the so called *Van Vleck correction*). For at least 5 bit quantization this implies a linear range for the signal RMS amplitude given by the formula $6(n_b - 4.2)$ dB, i.e. about 5 dB for a 5 bit quantization and 6 dB for each additional quantization bit.

In figure 1 the contributions to the nonlinearity of the r^3, r^5 and r^7 terms for a 8 bit ideal ADC are plotted as a function of the input signal RMS level, in ADU (green, red and cyan curves). As stated before, the dominant term is the first, and to achieve a linearity of 50 dB (first nonlinear term suppressed by 10^5 with respect to the actual correlated signal) the maximum RMS level is about 39 ADU, or 15% of the total ADC range.

2.1.2 Gain error

The quantization process slightly compresses the signal, due to truncation effects. This can be estimated by the linear term in the Taylor expansion of $R(r)$. In figure 1 (blue curve) the gain error (the linear coefficient minus 1) is also plotted, for an 8 bit ADC, as a function of the input signal level. This gain error is very low, less than 1% up to an input RMS level of 45 ADU, and can be calibrated together with other gain variations, e.g. in the receiver chain, as long as the input signal level remains constant.

If the input signal changes, however, the gain also changes, and a calibration error occurs. For example, if the calibration scale must be stable to -50 dB in presence of 1% variations of the signal level, the gain error should be below -30 dB (0.1%), that corresponds to an input level of about 37 ADU, sligthly less than the limit due to nonlinearities.

2.1.3 Quantization noise

An ideal quantizer introduces a rounding error, uniformly distributed between ±0.5 ADU. The added power is the variance of this distribution, 1/12 $(\mathrm{ADU})^2$. If the equivalent number of bits (ENOB) n_e is less than the actual bits in the quantizer n_b, the noise is proportionally increased to $2^{n_b-n_e}/12$.

The minimum signal amplitude depends on the requirements on the quantization noise. If the quantization noise must be N_q dB below the signal noise, this latter must have an input RMS level L given by the formula:

$$L = \frac{2^{n_b-n_e}}{\sqrt{12}} 10^{N_q/20} \tag{4}$$

where n_e and n_b are the equivalent and actual number of bits of the ADC, and N_q is referred to the spectrally averaged SNR.

Considering all these effects, the input signal level L for an 8 bit ADC must be comprised between this lower limit and about 37 ADU (14% of the ADC full scale).

2.2 Effects of RFI level on the digitization

If a deterministic signal (RFI) is present together with the stochastic noise, the overall ADC range is reduced, as the RFI changes the mean of the stochastic signal over time. The peak-to-peak amplitude of the RFI must be subtracted from the available ADC range, before applying the criteria stated above. For example for monochromatic RFI of RMS amplitude a_r (peak-to-peak amplitude of $2\sqrt{2}\,a_r$) this implies that the maximum RMS input level is $0.14(2^{n_b} - 2\sqrt{2}\,a_r)$.

This is valid for RFI composed of at most a few pure tones.

Most RFI signals today, either because of modulation or because they are composed of a large number of independent signals, behave like noise. For example an ensemble of 5-6 uncorrelated sinewaves with equal amplitude already show almost a Gaussian statistics. In this case, the above criteria must be applied to the total signal, (noise + RFI). For simplicity we will assume this situation, i.e. the ADC input level is set to the value in the previous section, considering the RMS amplitude of the total signal (RFI + sky + receiver noise).

2.3 Receiver gain instabilities and changes in signal level

The gain of the LNA, fibre and RX chain varies with time. The astronomical signal also varies with time but, due to the large instantaneous field of view of a single antenna element and the small intrinsic flux of individual sources, the expected variations are relatively small. The only significant source capable of changing the observed source level is the diffuse galactic emission. The brightness temperature of this emission, at around 150 MHz, varies by more than 14 dB between the galactic centre and the polar regions[8], but the emission averaged over the main lobe varies by about 6 dB. Long term gain fluctuations are expected to be smaller or comparable to those due to astronomical sources, with similar time scale, and short term gain fluctuations (e.g. $1/f$ component) are usually negligible on this scale.

Both these contributions to the signal level must be corrected by dynamically adjusting the signal before quantization. This adjustment can be performed on a relatively slow time scale. The adjustment step is about 1 dB, and it is reasonable to assume that the signal level can deviate from the optimum level by less than ±1 dB before the variation is corrected.

2.4 Spectral content of the digitized signal

The signal to be digitized has a nonuniform power spectrum, due both to the intrinsic spectrum of the astronomical signal and to the receiver chain spectral response.

A non-white spectrum decreases the SNR in the quantized signal, as the quantization noise is fixed with respect to the input RMS level, i.e. the spectrally averaged power density. If the spectrum is not white, some portions of the spectrum have a power density lower than the average, and the SNR in the corresponding channels of the channelized data is degraded.

On the other hand the SNR in the channelized data can increase with respect to the average if some portions of the input spectrum, that are not subsequently processed, are attenuated before digitization. These include the heavily RFI-contaminated portion below 50 MHz and the region affected by the anti-aliasing low-pass filter, above 350 MHz. In this case the quantization noise is spread over the whole Nyquist bandwidth, while the signal is concentrated in the narrower observed band. The gain is however modest, at most equal to the ratio between the Nyquist and signal bandwidths.

The astronomic signal is modelled as a power law, with spectral index $\alpha = -2.55$ and equivalent sky temperature of 60 K at 300 MHz, superimposed on the 4 K cosmic background:

$$T_s = 4\text{K} + 60\text{K}\left(\frac{\nu}{300\text{MHz}}\right)^{-2.55} \tag{5}$$

The receiver added noise is modelled as a frequency dependent component, 10% of the sky component, plus a constant term of 40 K.

$$T_r = \frac{T_s}{10} + 40\text{K} \tag{6}$$

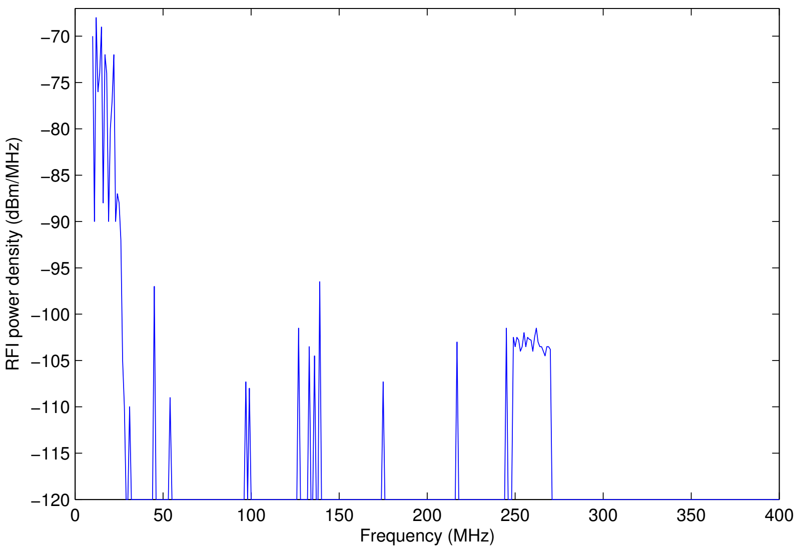

Figure 2: Observed RFI level (with 1 MHz channel bandwidth) at the site

Assuming a flat receiver chain response between 50 and 350 MHz, and a Nyquist frequency of 400 MHz, the average system temperature would be about 680 K, with a minimum value of 89 K. This would cause a degradation of the quantization noise SNR of 9 dB in the high frequency portion of the spectrum, with respect to the average.

It should be noted that, as long as the digital signal processing is correctly designed, there are no problems in dealing with portions of the spectrum with arbitrarily large spectral power densities.

The RFI contribution is not negligible. From the spectra observed in [6] (fig. 2), the largest spectral components are present below 50 MHz (page 22 therein). If these components are not attenuated, the integrated power below 50 MHz is about 23 dB above the integrated in-band power. An attenuation of at least 30 dB below 30 MHz is assumed to preserve signal integrity. The in-band RFI level is about 14 dB above the expected astronomical signal, for a spectral channel width of 1 MHz (page 29 of [6]). The affected portion of the spectrum is quite small, however, and the average power density is about 390 K, i.e. about 1/3 of the total signal seen by the ADC. The low frequency RFI, attenuated by 30 dB, contribute for another 20%.

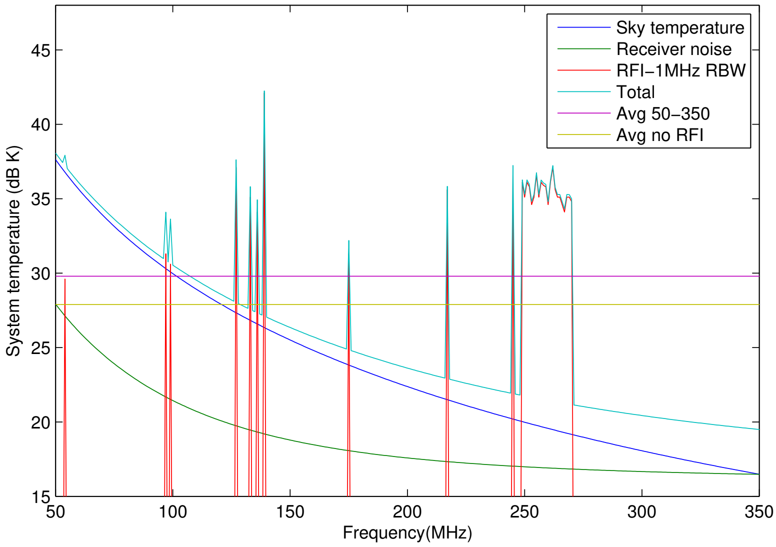

Figure 3: Spectrum of the quiet sky, the receiver noise, the RFI (with 1 MHz channel bandwidth) and average power density. Scale in dB(Kelvin)

All the signal components are plotted in figure 3, with a spectral resolution (for the RFI) of 1 MHz. The total spectrum as seen by the ADC, and the spectrally averaged signal level are also plotted.

The power density in the high frequency portion of the spectrum is about 15 dB below the average, with a corresponding increase of the quantization noise contribution. This is larger than previously calculated because the astronomical signal level has been reduced due to the RFI contribution.

2.5 LNA passband

The receiver chain is composed of a LNA, a RX block, and the ADC analog section. The total bandshape is dominated by the passband of the LNA, and of the antenna. In [5] the LNA gain, including mismatch effects, is reported. A reconstruction of the gain, in dB, with a spline interpolation, is shown in figure 4(blue). The contributions of the RX block is relatively flat, and is not considered for this analysis. This assumes that the LNA bandshape is representative of the total one. Also the total gain of the receiver chain is much higher, and must be adjusted in order to obtain at the ADC input a signal level as discussed in chapter 2.1.

The gain below 50 MHz is not reported in [5], and has been assumed to be about 30 dB below the average in-band gain, to properly attenuate the strong short-wave RFI. Above 350 MHz a reasonable low-pass anti-aliasing filter, with a rejection of ≈ 50 dB of the aliased band is also assumed.

The response is dominated by the actual response of the current LNA prototype, that is currently to be improved. The rolloff towards higher frequencies is likely representative of the final response, while the large gain dip around 60 MHz should be corrected. This gain curve, together with similar curves with a reduced gain dip, represents a useful testbench to investigate the effects of passband disequalization on the ADC performances. This will be discussed in section 3.

The same plot for figure 3 is repeated in figure 5, taking into account the gain shape of the LNA. The vertical scale has been normalized to the average power density. It can be seen that the large gain dip at 70 MHz causes the signal to drop 12.5 dB below the average level, i.e. slightly worse than the high frequency portion. According to the equation 4, to achieve a SNR of 200 (0.5% added noise) for the quantization noise of a 7.5 ENOB ADC, the input RMS level should be about 24 ADU.

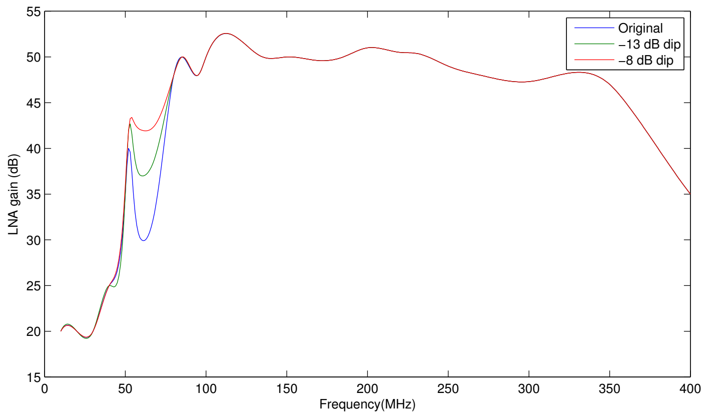

Figure 4: Gain of the LNA, in dB (from [5]). Gain outside the specification bandwidth (50-350 MHz) has been assumed from general system constraints. Alternate gain curves, with the dip at 60 MHz reduced to -13 and -8 dB have also been used

3 Quantization noise with and without equalization

All the effects previously discussed are then combined to derive an estimate of the quantization noise contribution as a function of the frequency.

The ADC noise is assumed flat, across the Nyquist band, with a total power equal to 1/12 the equivalent quantization step for the specified ENOB. For an 8 bit ADC with 7.5 ENOB, this corresponds to 1/6 $(\mathrm{ADU})^2$.

The input signal is assumed to remain within 35 ADU (chapter 2.1) at any time. This provides a small margin, with slightly degraded linearity, if the level rises momentarily to 38-40 ADU.

The signal level may vary for changes in source level, receiver gain fluctuations and RFI variations. The first two are slow and, as stated is section 2.5, it is assumed that it is possible to limit signal level fluctuations to ± 1 dB. RFI may vary rather quickly, and a margin of 5 dB (a factor of 3, or a factor ≈ 4 in RFI level if this contributes to 50% of the total in-band power) is assumed. Then the input level is set at 17.5 ADU (-6 dB with expect to the maximum), and it may vary from 16 to 20 (15.6 to 19.6) ADU. The digitizer noise is calculated for the worst case, i.e. 16 ADU of RMS input level.

The LNA band shown in section 2.5, with a peak-to-peak disequalization of 23 dB, is clearly problematic, and different values for the dip around 60 MHz have been tested (fig. 4). In this way the effect of a nonuniform spectral response is estimated for decreasing values of the RX chain peak-to-peak disequalization.

A simple equalization scheme would produce a slope of 6 dB/octave, almost cancelling the $\alpha = -2.55$ spectral slope. A simple high pass network, with a pole at 400 MHz, has been assumed. A more complex filter could be required to guarantee proper matching to the receiver.

3.1 Original LNA passband

Both situations, with and without the equalization network, have been plotted in figure 6 for the original passband (23 dB LNA peak-to-peak diseqalization).

The specification for the spectrally resolved quantization noise is shown in red. As seen in section 2.5, the quantization noise for the non-equalized signal exceeds the specifications by about 3-4 dB around 60 MHz, and by a couple of dB above 270 MHz.

The equalization completely solves the situation in the high portion of the band, but seriously aggravates it around 60 MHz, to more than 10 dB with respect to the specifications (about 5% added noise).

The contribution of the RFI to the total power is about 55%, 19% due to the out-of-band RFI and 36% to the in-band component. The equalization changes a bit the contribution, with 62% of total RFI, almost all due to the in-band component.

The equalization filter, being a passive network with strong attenuation of a significant portion of the spectrum, significantly reduces the signal level. The RMS level after the equalization is attenuated by almost 10 dB, that must be recovered by increasing by 10 dB the overall gain (e.g. in the last amplifier before the ADC).

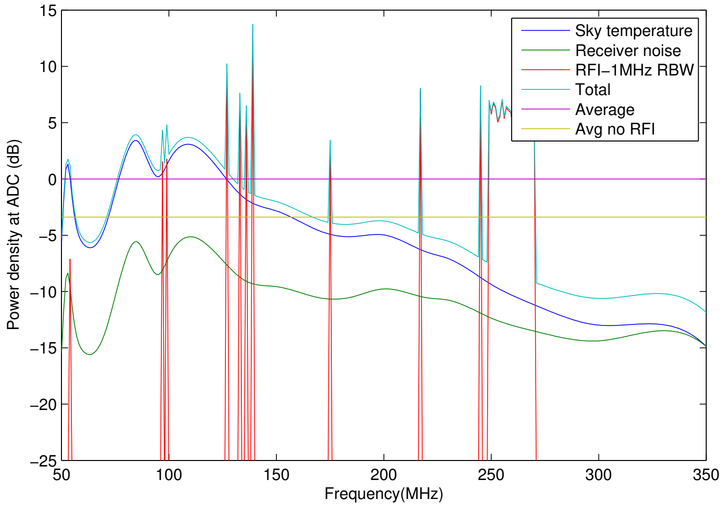

Figure 5: Spectrum of the signal components of fig. 3 considering the measured LNA gain. Scale in dB with respect to the average input level

3.2 Improved LNA pass-bands

By reducing the dip at 60 MHz to -13 dB, the non equalized signal satisfies the quantization noise specs up to 260 MHz.

The quantization noise in the high portion of the spectrum remains below 1% of the total noise (figure 7, blue curve). The noise specification could be satisfied by increasing the signal level at the ADC input, at the expense of reducing the margin for an increase of the RFI level.

With the equalization network (figure 7, green curve) the region around 60 MHz returns below the specifications, even if not so seriously as before. To keep the quantization noise below the specification, the dip must be reduced to 8 dB (red and cyan curves), with a peak-to-peak receiver gain disequalization of about 12 dB.

3.3 Problems due to equalization

The equalization noise, attenuating most of the spectrum by up to 20 dB, requires a further amplification of the signal by ≈ 10 dB. This may introduce possible problems of nonlinearities, increases the power for the amplifier, and the overall system complexity.

As most of the RFI present at the sites are around 200 MHz, the integrated RFI power increases, with respect to the astronomical signal, by about a factor of 2, becoming the dominant component in the digitized signal. This makes the system more vulnerable to a sudden increase in RFI power. On the other side, the equalization network helps suppressing the strong RFI present below 30 MHz.

4 Effects of a spectral slope in the receiver response

The antenna calibration is performed on relatively wide frequency channels, for sensitivity reasons. The underlying assumption is that the received noise across one channel is relatively flat, so that the calibration can be attributed to the whole channel. This is especially important for the low end of the spectrum, where the fractional bandwidth of one channel can be quite high, in the range of a few percent.

If the power density varies significantly across one channel, the calibration (especially the phase calibration) does not refer to the nominal channel centre, but is offsetted in frequency. If the channel width is Δf, and the gain slope is dG/df (in dB per Hz), the frequency offset δf with respect to the nominal channel centre f_0 is approximately

$$\delta F = \frac{1}{12} \frac{\log(10)}{10} \frac{dG}{df} (\Delta f)^2 \qquad (7)$$

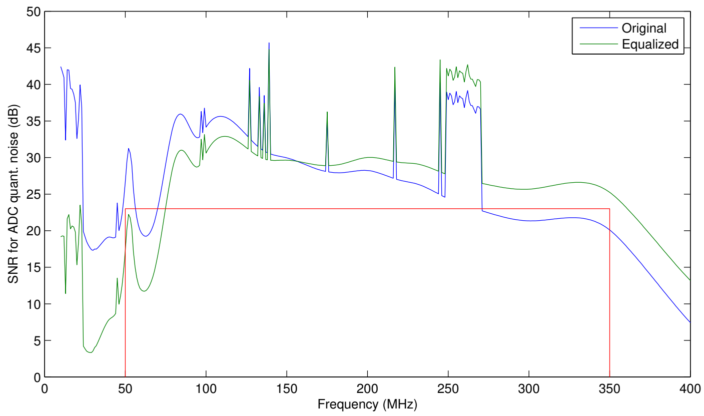

Figure 6: Spectral density at the ADC, normalized to the quantization noise. Orignial LNA bandshape. Specification of 23 dB for the in-band SNR is outlined in red

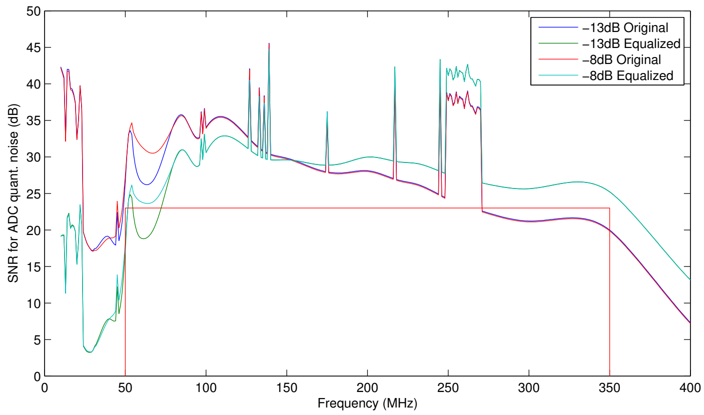

Figure 7: Spectral density at the ADC, normalized to the quantization noise. Receiver gain curves with -13 dB and -6 dB disequalization at 60 MHz

The visibility phase depends on the geometric delay, that in turn depends on the baseline length. A fractional frequency error, $\delta f/f_0$, directly translates to a relative baseline error $\delta B = B\delta f/f_0$. For a celestial source near the edge of the field of view, if the baseline error becomes comparable to the station radius, nominally 17.5 m, the signals for different baselines across the array will decorrelate, reducing the array sensitivity and then the maximum instantaneous field of view. For example having a baseline error of 10 metres reduce the field of view from the nominal station beam of 10 degrees at 50 MHz to about 7 degrees. Reducing the baseline error to 5 metres still cause a degradation of the field of view to 8.5 degrees. The maximum gain slope is then:

$$\frac{dG}{df} = \frac{120}{\log(10)} \frac{f_0}{(\Delta f)^2} \frac{\delta B}{B_{max}} \tag{8}$$

Assuming a maximum baseline error of 10 m, for the longest LFAA baseline, 100 km, and at the frequency where the gain variation is the steepest, aroud 50 MHz, the maximum $\delta f = f_0 \delta B/B = 5kHz$, that in turn limits the gain slope to 0.3 dB/MHz. This requirement is reduced at higher frequencies, but as the instantaneous field of view is also lower, the degradation is more important and it is safe to assume this at any frequency.

For comparison the slope in the gain curve shown in figure 4 is greater than 1 dB/MHz. This corresponds to a baseline error of about 35-40 metres at 50 MHz, i.e. larger than the baseline radius, reducing the field of view to less than 4 degrees.

In principle it would be possible to compensate these errors by an accurate measurement of the receiver gain curve, but the method is difficult to apply to a number of receivers of the order of half a million and a likely source of systematic errors.

5 Conclusions

An 8 bit digitizer, with 7.5 ENOB, is adequate for the LFAA signal chain.

RFI contributes with an integrated power comparable to the astronomical signal. As RFI intensity could change on a short time, 5 dB are reserved for possible RFI fluctuations. Setting the attenuator for a goal RMS level of 17.5 ADU and adjusting periodically the signal level, this should stay between 14 and 35 ADU in all circumstances.

An RMS signal level of 16 ADU produces a spectrally averaged SNR within the assumed specifications with a 9 dB margin. Therefore the signal, as seen by the ADC, must be equalized in frequency within ± 9 dB. The actual signal, including the RFI contribution, is disequalized by up to 12 dB. An equalization network could limit the signal disequalization to ± 3 dB, leaving ± 6 dB for the receiver gain flatness.

Fine tuning of the gain control algorithm, considering the variability of the RFI and signal situations, could allow a slightly higher signal level at the ADC input, improving the performance both with and without the equalization network.

To ensure a reliable calibration, the spectral slope cannot exceed 0.3 dB/MHz. Higher values would produce a significant reduction in the instantaneous field of view, due to the induced errors in the calibration procedure.

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Contents

List of Figures